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Proposed Deterministic CNOT Gates Based on Coherent Photon Conversion

A Thesis

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By

Jawad A.K. Hasan

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Supervisor: Prof. Dr. Khalil Hajim

Certification

I certify that this thesis was prepared under my supervision at the Institute of Laser for Postgraduate Studies, University of Baghdad as a partial requirements for the degree of Doctor of Philosophy in Laser/ Electronics & Communication Engineering.

Signature
Name: Dr. Khalil Ibrahim Hajim
Title: Professor
Address: Institute of Laser for Postgraduate Studies,
University of Baghdad
Date: / /2012

In view of the available recommendations, I forward this thesis for debate by the examination committee.

Signature
Name: Dr Abdul Hadi M. Al-Janabi
Title: Professor/ Head of the Scientific Commitee
Address: Institute of Laser for Postgraduate Studies,
University of Baghdad
Date: / /2012

وَيَسْأَلُونَكَ عَنِ الرُّوحِ ۖ قُلِ الرُّوحُ مِنْ أَمْرِ
رَبِّي وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

صدق الله العلي العظيم

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To

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To

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To

my daughter **Shahd** and my sons **Ameen** & **Mustafa**.

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LIST OF SYMBOLS

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symbol	*meaning*
$\hat{a}$	*Annihilation operator*
$\hat{a}\dagger$	*Creation operator*
$\hat{A}$	*Hirmetian operator*
B_8	*Three qubit eight computational basis*
B_3	*Three qubit five computational basis*
B_p	*Polarization basis*
B_c	*Computational basis*
$\mathcal{B}$	*Number statebasis*
$\hat{H}$	*Hamiltonian operator*
H	*Vertical polarization*
$\mathbf{H}$	*Hilbert space*
$\hat{n}$	*Number operator*
V	*Vertical polarization*
$\|n\rangle$	*Number state*
$\|\alpha\rangle$	*Coherent state*
U	*Unitary operator*
λ	*Eigenvalue*
γ_1	*2nd order coupling strength*
γ_o	*3rd order coupling strength*
Γ	*Interaction coeficient*

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LIST OF ABBREVIATION

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CNOT	*controlled NOT gate*
CCNOT	*controlled controlled NOT gate*
CZ	*controlled Z gate*
SWAP	*swapping gate*
QM	*quantum mechanics*
qubit	*quantum bit*
cbit	*classical bit*
CPC	*coherent photon conversion*
CT	*control target qubits*
LOQC	*linear optical quantum computing*
FWM	*four wave mixing*

Abstract

Photonic CNOT gates are considered as one of the main building blocks of a quantum computer. They are excellent sources for entangling photonic qubits but current schemes for implementing CNOT operations are not efficient to fulfill the requirement for a scalable photonic quantum computation. In this thesis two mathematical CNOT operators have been devised for the first time and verified by carefully selecting the logical bases, then photonic circuits for implementing deterministic CNOT operations have been investigated. They are based on the quantum optics mechanism of coherent photon conversion (CPC) which was recently only used for the construction of controlled Z (CZ) gates. The first design architecture consists of two identical CPC components that operate on general three single mode states distributed among an ancilla, a control and a target. This scheme has less complexity and can be realized in the computational Fock bases ($|0\rangle$ and $|1\rangle$) with two sections of CPC and some linear optical elements. The second design architecture is a dual rail scheme with three CPC components . It has less complexity also and can be realized in the polarization basis (H,V) with three sections of CPC and some linear optical elements.

Since the pump field is a photonic coherent state, therefore, it can be used as a switching parameter for the gate function to minimize the processing time with respect to the decoherence time.

Contents

1 Ch.1: Introduction and Basic Concept

1.1 Introduction

Historically, it can be said that the computing has been started when human began counting numbers with his fingers, pebblels and clay until he invented the methods of writting. Representing numbers by symbols had a real impact on computing development and motivated the inventions of different tools to perform arithmatic operations. Abacus was one example of the tool used as a hand calculator. It was invented in Babylon about 2400 BC. The first style of abacus was made of lines drawn in sand with spheres of pebbles. The more modern abaci designs, are still used today as tools of calculation for primary schools . This was considered as the first known computer and most advanced ancient calculating system known to date which preceded Greek methods by 2000 years [1].

In the Islamic world, mechanical analog computer devices appeared again and were developed by Muslim astronomers, such as the equatorial astrolabe by Abu Rayhan Al- Biruni. Muslim engineers have also invented programmable machines , such as the automatic flute player by the Banu Mosa brothers [1].

Algorithm was the analytical concept of computation, which is a procedure for constructing a solution to a certain problem. The word algorithm was first used in the 9th century after the mathematician, Abu Abdullah Muhammad bin Musa Al-Khwarizmi, who published several scientific books. His famous book, Al Jabr wa al Muqabilah contains procedures for solving specific kinds of linear and quadratic equations which were studied in Europe for more than 500 years. The word, algebra, was named after its title . The increasing requirement to perform complicated calculations in the the second millennium resulted in the developments of mechanical devices. Infact the natural development in different calculating tools aimed to intiate a set of calculations which were used to solve the arithmetical problems [1].

In 1831 Babbage designed an analytical engine which was the first approach for a general purpose programmable computer. This design used steam to feed a control system of mechanical gear box that processed programs which was encoded onto punched cards. The output would be taken as a text from a printer or graphics from a plotting device [1, 2].

Many computational models have been formalized in the first half of the 20th century until Turing showed in his famous that the computational models, including the Universal Turing Machine (UTM), were compatible. This model of computation is still considered as the most practical model of computation [3] .

Algorithms were classfied according to their required amount of computational resources required. Time is an important resource for a computer. The other important resource is space, which denotes to the amount of memory used by the computer in performing the computation. The measure used for the amount of a resource in a computation for solving a given problem is a function of the length of the input (number of bits) of an instance of that problem. "For example, if the problem is to multiply two n bit numbers, a computer might solve this problem using up to $2n^2 + 3$ units of time (where the unit of time may be seconds, or the length of time required for the computer to perform a basic step) [4].

From 1960 and on, computational complexity formalization began ,resulting in a new field of computer science depending on the steps used in solving the computational problems [8 -11].
An algorithm is cosidered to be efficient if its running time function do not grow faster than some polynomial in the input size n, and inefficient if it

grows faster than any polynomial. Accordingly a computational problem is classified as easy or hard if there exists an efficient algorithm to solve it, and hard if no such algorithm exists. In application, a wide range of algebraic, engineering and other problems are turned to be easy by simulating an efficient algorithm for solving them. For problems that appear to be intrinsically hard, the only way we can solve large instances of hard problems is to build massively powerful computers, and to design a powerful and a fast computation algorithm [4].
Digital computers are considered as the first and only practical, general purpose reliable and scalable computing device . Its invention harnessed the computing power to be more significant that it was considered as the transition point in history between the Industrial Age and the Computing Age. The development in information theory, coding theory, and computer engineering motivated digital computing to be more reliable [1].

Digital Circuit Model suggested by Shannon in 1937 , showed that the Boolean algebra system can be implemented using readily available electronic switches [9]. Nine years later, von Neumann built the so called von Neumann Architecture which provided a basic framework for the of scalable and general purpose digital computers design using digital circuits [1]. The invention of error correcting digital circuits which resulted approximtely in 1950 was considered as the breakthrough in coding theory. These techniques helped to build highly reliable computing devices from less reliable parts [13-15].

The main revolution in in computer engineering was the invention of the microprocessor which was began after the invention of the transistor. Transistors were immediately becoming the main elements in building digital computers since it could be used to affect the size, reliability, and the cost of the digital circuit. At first transistor did facilitate several early digital computers, but the most effective thing was the microprocessor by fabricating of thousands of trasistors on a small chip area which was the real impact on digital computer [1].

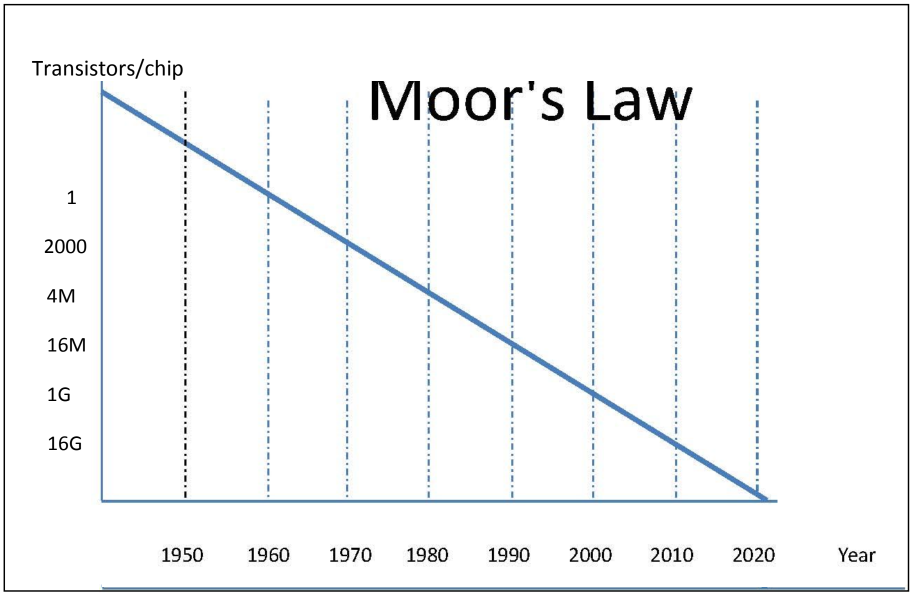

Figure 1: **Moore's Law[13]**

In1965 Moore observed that the number of transistor in the typical microprocessor was doubling approximately every 18-24 months as shown in Fig.1[13]. His prediction was then known as Moores Law which showed that the size of transistors as a function time will be decreased. If this rate continued at the given rate, physical systems which encode classical bits of information will be in of the size of atoms in approximately the next 10 years at which, the quantum effects will be dominant. To confined all these developments and explosion of computing power, hardware and software devices were required. Moores law, refers to the time when transistors sizes will be decreased to the atomic scale at which the computing devices will enter the domain of quantum mechanics [16-18].

1.2 Classical Logic

The classical computation theory began when Church and Turing have successfully and independently developed a classical discrete computation model given by [3, 17];

$$x_1, x_2..., x_n \xrightarrow{f(x_1, x_2..., x_n)} b \tag{1}$$

where $b = f(x_1, x_2..., x_n)$ is single-valued function on n discrete inputs and this function has been assumed to be simulated and computed by physical model. This function was known as n-bits algebraic function where the input variables $(x_1, x_2..., x_n)$, and b are binary variables, or bits, of values, 0 or 1. In this case, the function $f(a_1, a_2..., a_n)$ is known as an n-bit Boolean function. The main problem concerned any efficient computer is the universality, which means that any function whatever large can be simulated on its inputs by a universal set of operations called gates which repeatedly used in acertain fashin according to the type of the function f. Classical computation can be done with logical gates, AND, OR, and NOT gate, which are considered as universal gate for classical computation, since it is possible to simulate any function of the form illustrated in Eq.(1) by combinations of their operations. Classical logic gate funtions were allway given by Truth tables as in table (1) [15, 16]. Any Boolean functions can be simulated with a set of AND, OR, and NOT gates. So these three gates are universal for classical Boolean logic. To simplify manufacturing and fabrication of logic circuits the three aforementioned gates were simulated by repeatedly operations of specific gate called NAND gate as given by [15, 16]:

$$a\ NAND\ b = NOT(a\ AND\ b) \tag{2}$$

Table 1: **The logical relation of classical logic gates** AND, OR, $\&NOT$.

AND	OR	NOT
a_1 a_2 b	a_1 a_2 b	a b
0 0 0	0 0 0	0 1
0 1 0	0 1 1	1 0
1 0 0	1 0 1	
1 1 1	1 1 1	

The three mentioned universal gates can be simulated by NAND operations such that

$$\begin{aligned} a\,OR\,b &= (a\ NAND\ a)\ NAND\ (b\ NAND\ b) \\ a\ AND\ b &= (a\ NAND\ b)\ NAND\ (a\ NAND\ b) \\ NOT\ a\ &= a\ NAND\ a \end{aligned} \quad (3)$$

Therefore it is important here to refer that the two-bit NAND gate alone is sufficient for classical Boolean logic. The number of NAND gates required to simulate a function with n inputs is exponentialy proportional to n since NAND gate is a two bit input and only one bit output function. where one of its inputs has effectively been lost in the process, whose information has been irretrievably lost which makes an entropy change of $ln2$ for every bit lost and this amounts to an energy increase of kT ln2 , where k is Boltzmans constant and T is the temperature in Kelvin [18, 19]. This loss will cause heat dissipation which is taken to be an indication of physical irreversibility [19]. To overcome this problem Bennett in 1973 proposed reversible physical operation which then used in designing reversible classical computation model that can be done with no energy dissipation [20] .

It was shown that reversible model can be effeciently used to simulate any problem that can be simulated on the original irreversible machine.Therefore the computing machine implemented by physical reversible model implied that the logical operation done by this model are also reversible. This debate intiated anew field of research aimed to build physical models for reversible classical computation [20]. In 1981 Toffoli has been succeded to identify three bit universal gate called the Toffoli Controlled-Controlled-NOT (CC-NOT). He showed that this three-bit gates are universal for classical reversible computing. The Toffoli CCNOT gate is given by Eq.4 in the matrix representation of the three bit orientation (000, 001, 010, 011, 100, 101, 110, 111). The most significant consequence of this gate is that Toffoli gate and other three bit classical reversible gates are universal for reversible computing contrary to the two bits reversible gates [21].

$$CCNOT = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{4}$$

1.3 Quantum logic development

For the years between 1973-1980, all efforts have been directed to make classical computing done in reversible fashion to decrease the energy dissipation. The first ideas about using quantum models for computation started by Fynman [22, 23]. At first, most researches in the field of quantum computing were mainly done for academic purposes to explore the capabilities of quantum computing. It was Peter Shor who gave the motivation of the quantum computing , when he announced his quantum algorithm for factoring large numbers in 1994 with an efficiency unparalled by any classical algorithm preceding it [24]. This paper was considered as the real pay off for the quantum computation, where factoring problems were widely used in public key cryptography to encrypt messages, which added to the quantum computing an urgent tasks. The most effective advancing in quantum computing has come in 1985 by Deutsh when he showed that there exists a quantum analog to the Turing machine which was considered as a landmark paper on this subject. Deutch assigned two important issues to build a computer model, the storage unit and processing gates which concerns to type of the memory units of quantum computer, and how to process the information contained in them to do the computation [15, 25].

1.4 Quantum mechanical notes

Classical or Newtonian mechanics is used to describe classical physics, while quantum mechanics is the mathematical formulation of quantum physics [4]. Therefore, it is useful to present here a short note about relevant quantum mechanical concepts of relevance to the operations of quantum computing.

State space is always used in quantum mechanics to describe all the known properties of that state. It differs from that of classical mechanics in the ability of quantum systems to exist in coherent superposition of states with complex amplitudes. The other differences relate to the tensor product description of multiparticle systems. Therefore complex vector space was the best choice for the quantum mechanical state space [15] .

The basis is defined as set of vectors $|x\rangle_1, ..., |x\rangle_k$ in the vector space V (in Dirac bra-ket representation) and satisfies [26]:
a)$|x\rangle_1, ..., |x\rangle_k$ are linearly independent.

b)$\forall |x\rangle \in V \; \exists \; \lambda_i \in C$

such that

$$|x\rangle = \sum_{i=1}^{k} \lambda_i |x\rangle_i \tag{5}$$

1.4.1 Hermitian Operators

" Any physical quantity (observable, e.g. position X) is associated with an orthonormal basis $|x_i\rangle$ (the observables eigenbasis) whose elements (eigenstates or eigenvectors) correspond to possible values (eigenvalues) of the observable. So to measure the observable,it has to construct a measurement apparatus associated with the observables eigenbasis . After the measurement, the apparatus will display the value corresponding to the basis element onto which the state has been projected. Every observable and the apparatus, is associated with an operator, defined as [27],

$$\begin{aligned} \hat{X} &= \sum |x_i\rangle\langle x_i| x_i \\ \hat{X}|x_i\rangle &= x_i|x_i\rangle \end{aligned} \tag{6}$$

Every physical observable is measurable and entirelly has real eigen values which represent the measurement values of that observable, therefore if the system is prepared in an eigenstate of an observable, measurement of this observable will always (with probability one) yield the corresponding eigenvalue. "Associating operators with observables appears useful since these operators carry full information about this observable and are also useful in calculations. For example, they provide statistical information about possible outcomes of measuring the observable [27]".

The operator $\hat{A}$ is called Hermitian operator or self-adjoint if [15]

$$\hat{A} = \hat{A}^{\dagger} \tag{7}$$

For Hermitian operator all eigenvalues of $\hat{A}$ are real and the eigenvectors corresponding to different eigenvalues are orthogonal [15].

The eigenvalues of a matrix are basis independent . Therefore it is suitable to represents a physical observable with the measurement outcomes to identify the eigenvalues of the associated observable. In this case this required that the operator has only real eigenvalues and this can be satisfied only with Hermitean operators [26].
On an N- dimensional Hilbert space, Hermitean operators are often represented by an $N \times N$ matrix which have N eigenvalues, denoted by λ_i corresponding to the eigenvectors $|\lambda_i\rangle$. If the eigenvalues of the Hermitian matrix are all different, then the corresponding eigenvectors are orthogonal. Therefore the eigenvectors form a set of N pairwise orthonormal vectors in an N-dimensional Hilbert space. Therefore any Hermitean operator $\hat{A}$ can be expanded in its eigenvectors and eigenvalues which is called diagonalization as [15]

$$\hat{A} = \sum_i \lambda_i |\lambda_i\rangle\langle\lambda_i| \tag{8}$$

To represent this matrix in other basis, it is important to find a map between the original basis (i.g canonical basis $|e_i\rangle$) and the orthogonal basis of the eigenvectors of $\hat{A}$ since thee matrix form of an operator $\hat{A}$ is basis dependent..

1.4.2 Unitary operators

An operator matrix $\hat{U}$ is called unitary if it is defined for all vectors $|x\rangle, |y\rangle$ in the Hilbert space $\mathbf{H}$, such that [15]:

$$\langle x|\hat{U}^\dagger \hat{U}|y\rangle = \langle x|y\rangle. \tag{9}$$

The above condition can be replaced by:

$$\hat{U}^\dagger \hat{U} = \hat{U}\hat{U}^\dagger = 1 \tag{10}$$

Eq.(9) implies that a unitary operator preserves the scalar product and therefore the norm of vectors.

Any unitary operator $\hat{U}$ on an N-dimensional Hilbert space H has a complete basis of eigenvectors and all the eigenvalues are of the form $e^{i\phi}$ with real ϕ i.e [15],

$$|\lambda^2| = 1 \Leftrightarrow \lambda = e^{i\phi} \tag{11}$$

To any unitary operator $\hat{U}$ there is a Hermitean operator $\hat{A}$ such that [26]

$$\hat{U} = e^{i\hat{\hat{A}}} \tag{12}$$

Any unitary operator $\hat{U}$ associated with the Hermitean operator $\hat{A}$ can be represented by the eigenvalues a_k and eigenvectors $|a_k\rangle$ of $\hat{A}$ such that [26]:

$$\hat{U} = e^{i\hat{A}} = \sum_{i=1}^{N} e^{ia_k}|a_k\rangle\langle a_k| \tag{13}$$

This is an operator which has eigenvalues of the form e^{ia_k} with real a_k. The above two relations are very useful in the Hamiltonian analysis and in the computation of the eigenstates and eigenvalues . It can also be verified that for any unitary operator, the following relation is satisfied [28, 29]

$$f(\hat{U}^\dagger \hat{A} \hat{U}) = \hat{U}^\dagger f(\hat{A}) \hat{U} \tag{14}$$

1.5 Quantization of Light

With low levels light , where the number of photons is small, the quantum theory of light is very important. In this case the fluctuation around the avarage field value $1/\sqrt{N}$ is not as small (where N is the average number of photons) as in the classical limit and hence the fields cannot be considered to be continuous. Starting from Maxwells equations, the genralized coordinates and their derivatives in the QM system must have a commutation relation defined analogous to that of harmonic oscillator such as [30]

$$[\hat{x}, \hat{p}_x] = xp_x - p_x x = i\hbar \tag{15}$$

where x and p_x are the position and momentum operators respectively. Here the quantized EM field is considered to be equivalent to a system of infinite set of harmonic oscillators [32]. As a result of the field quantization, the basic quanta of the EM fields, which are called photons, are created and annihilated in discrete processes of emission and absorption by atoms or matter in general [30].

The Hamiltonian of a single mode EM is [32]:

$$\hat{H}_k = \frac{\hbar\omega}{2}(\hat{a}_k \hat{a}_k^\dagger + \hat{a}_k^\dagger \hat{a}_k) = \frac{\hbar\omega}{2}(\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2}) \tag{16}$$

where $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the creation and annihilation operators of the k_{th} mode respectively.

1.5.1 Number states

As in the harmonic oscillator, a photon number eigenstate $|n\rangle$ is defined as an eigenstate of a photon number operator $\hat{n}$ with an eigenvalue n [33]

$$\hat{n}|n\rangle = n|n\rangle \tag{17}$$

The graund state (vacuum state) $|0\rangle$ is defined by [33]

$$\hat{a}|0\rangle = 0 \tag{18}$$

$|0\rangle$ is a number state of a smallest eigenvalue and $\hat{a}$ is a photon annihilation operator represented by [33]:

$$\hat{a} = \frac{1}{\sqrt{2\hbar\omega}}(\omega\hat{q} + i\hat{p}) \tag{19}$$

and substituting in Eq.(18) the relations,
$\hat{q} = x$ and $\hat{p} = -i\hbar\frac{\partial}{\partial x}$
the following relation is obtained [30]:

$$\langle x|\hat{a}|0\rangle = \frac{1}{\sqrt{2\hbar\omega}}(\omega x + \hbar\frac{\partial}{\partial x})\psi_0(x) = 0 \tag{20}$$

where $\psi_0(x) = \langle x|0\rangle$ is a Schrodinger wavefunction of a vacuum state in x representation.
The solution of Eq.(20), results in [30]:

$$\psi_0(x) = (\frac{\omega}{\pi\hbar})^{\frac{1}{4}} e^{[-\frac{1}{2}(\frac{\omega}{\hbar})x^2]} \tag{21}$$

Eq.(21) shows that the vacuum state is a Gaussian wavepacket centered at x= 0 and with a variance of $\frac{1}{2}\frac{\hbar}{\omega}$.
The ground state energy of the radiation field is given by [33],

$$H_{rad}|0\rangle = \frac{1}{2}\sum_k \hbar\omega_k|0\rangle \tag{22}$$

The sum diverges, since in the summation over the modes, corresponding to unbounded frequencies is not bounded from Eq.22. An interesting consequence of the quantization of the radiation field is the fluctuations associated with the zero - point energy or the so-called vacuum fluctuations . These fluctuations have no classical analog and are responsible for many interesting phenomena in quantum optics [32].

A number state with a higher eigenvalue may be obtained by [30]

$$|n\rangle = \frac{1}{\sqrt{n!}}(\hat{a}^{\dagger})^{n}|0\rangle \tag{23}$$

and corresponds to the wave function [30]

$$\psi_n(x) = \langle x|\frac{1}{\sqrt{n!}}(\hat{a}^{\dagger})^{n}|0\rangle \tag{24}$$

The photon number eigenstates form a complete orthonormal set since they are eigenstates of a Hermitian operator $\hat{n}$:(the eigen states of a Hermitian operator form a complete set) or[26]:

$\sum_n |n\rangle\langle n| = 1$

1.5.2 Coherent states

A coherent state $|\alpha\rangle$ is defined as an eigenstate of a photon annihilation operator $\hat{a}$ with a complex eigenvalue α,[34, 36]

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \tag{25}$$

For $\alpha = 0$, Eq.(18) will be obtained. Thus from Eqs.(25,18), the vacuum state $|0\rangle$ is simultaneously a photon number eigenstate and a coherent state. The coherent state $|\alpha\rangle$ can be expanded in terms of photon number eigenstates as [37]:

$$|\alpha\rangle = \sum_n |n\rangle\langle n|\alpha\rangle \tag{26}$$

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle \tag{27}$$

Gaussian wavepacket of a coherent state $\psi_\alpha(x)$ can be derived simillar to that of Eq.(20) [30].

$$\psi_\alpha(q) = ((\frac{\omega}{\pi\hbar})^{\frac{1}{4}} e^{[-\frac{\omega}{2\hbar}(q-\langle\hat{q}\rangle)^2 + \imath\frac{\langle\hat{q}\rangle}{\hbar}]q} \tag{28}$$

The coherent and vacuum states have identical variances $\langle\hat{q}^2\rangle = \frac{1}{2}\frac{\hbar}{\omega}$. The measurement statistics of a coherent state for a photon counting measurement is given by [37]:

$$P(n) = (\langle n||\alpha\rangle)^2 = \frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!}, \tag{29}$$

which is a Poisson distribution with an average photon number $\langle n\rangle = |\alpha|^2$ [38]. A coherent state is a minimum uncertainty state where $\Delta p\Delta q = \frac{\hbar}{2}$ and Coherent states form a complete set i.e $\frac{1}{\pi}\int |\alpha\rangle\langle\alpha| d^2\alpha = 1$ [37].

1.5.3 Thermal States

Thermal states are the third class of quantum states of light.They are produced by thermal sources, like light bulbs. The photon statistics of a thermal state is given b Boltzmann distribution:
$p(n) = \frac{1}{1+\bar{n}}(\frac{\bar{n}}{1+\bar{n}})^n$ [30]

1.6 Quantum Information

Quantum information, in general has been developed by the efforts of many researchers, examples R. Landauer in the 1960s studied the thermodynamic effects of irreversible operations in computation and showed that every rirreversible operations was associated with loss of energy and by using reversible operation, C. Bennet showed that the thermodynamical effects can be reduced [40, 41]. Quantum measurements which are irreversible operation were explained by J. von Neumann , C. Helstrom and A. Holevo who investigated the bounds on the capacity of the quantum communication channels when they are used to transmit classical information [45-48]. In the early works, the efforts concentrated on the limitation of classical information processing and to show the advantage of using quantum mechanical computing and its application in quantum cryptography which was introduced by C. Bennett, et al [41].

Quantum computer was first introduced in the 1980s, by R. Feynman and P. Benioff [46]. They showed that the complexities inherent in a classical computer might be reduced by using a computer based on quantum mechanics. The quantum computing issue stayed in the theoretical researches until 1994 when P. Shor published his algorithm for factoring large whole numbers which was considered as a rea pay off for building more effective quantum cryptographic protocols [24] .

1.6.1 Classical Bit (cbit) and Quantum Bit (qubit)

The basic unit of information of a classical discrete computer are bits, which may be a two-valued classical entities, example, in TTL logic, the zero and five voltage levels are used to represents bit of logic 0 and 1 respectively . By analogy, quantum bit can be simulated by a two-level quantum system, or eigenstates labeled $|0\rangle$ and $|1\rangle$. This representation corresponds to the classical bit, 0 and 1 respectively. Qubits can be simulated by photons in the polarization basis or in the number state basis , electrons and other spin- 1/2 systems [39].

Single qubit systems are very important in quantum computing since all other complex systems can be modeled with multiqubits in the Hillbert space. Qubit is a two eigen state , and can be in a superposition state $\alpha|0\rangle + \beta|1\rangle$ also that is, in addition to $|0\rangle$ and $|1\rangle$, with [39]:

$$\alpha^2 + \beta^2 = 1 \tag{30}$$

where α and β are complex numbers called the probability amplitudes of the superposition.

The superposition princible of quantum states is one of the important properties of quantum theory. In summary the classical bit is a two valued amplitude encoding system while qubit is a continueous phase encoding system [16]. Qubits can be represented by Bloch sphere Fig.2 where the eigen states $|0\rangle, |1\rangle$ lies on the Z axis of unit sphere which related to Z-measurement, while the states $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ lie on the X axis called Hadamerd basis. The quantum state $|\psi\rangle$ has norm one and angle θ, ϕ with Z and X axis respectively [19].

$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\phi}sin\frac{\theta}{2}|1\rangle \tag{31}$$

The quantum effect of the superposition principle will extend the number of processing states [39].

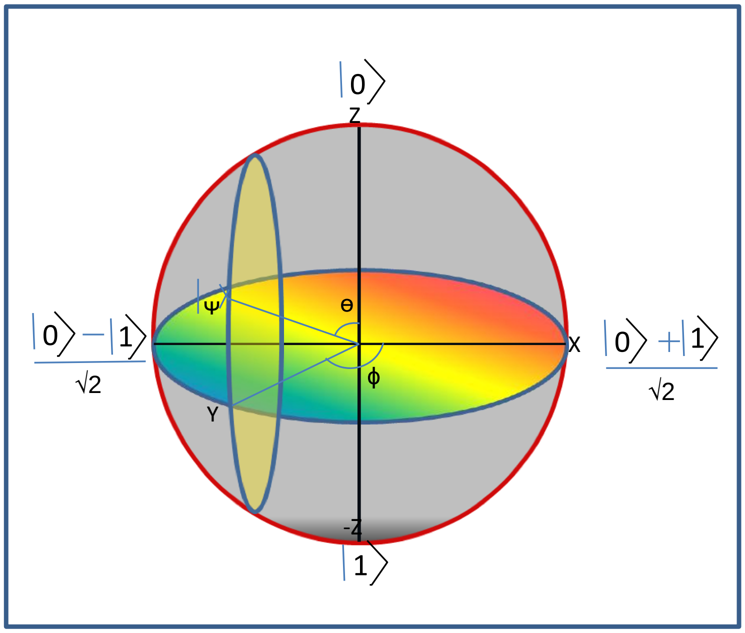

Figure 2: **Bloch sphere representation of quantum bit. The eigen states $|0\rangle, |1\rangle$ lies on the Z axis of unit sphere which related to Z- measurement, while the states $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ lie on the X axis. The state $|\psi\rangle$ has norm one and angle θ, ϕ with Z and X axis respectively**

1.7 Multiqubit systems

Multi-bits classical system state exists as strings of bits, while n qubits system is described by the tensor product of n single qubit states. If there is more than one qubit in the quantum system, product eigenstates are used to express its state. The ability of a quantum system to live in a coherent superposition of many eigenstates will create the quantum interference between all eigen states which is very important in quantum computation and facilitates the quantum computer to deal simultaneously with all eigenstates which has no classical analogy. For example, a two-qubit system has the basis states,$|00\rangle$,$|01\rangle$,$|10\rangle$,$|11\rangle$ [39]

The classical states constructed from two classical bits have the possibilities to be in one of the four states (00,01,10 or 11) but two quantum bits could be in a coherent superposition of all four states, while three qubits could be in a superposition of eight states ($|000\rangle$, $|001\rangle$, $|010\rangle$, ...$|111\rangle$) .This is the main distinction between classical and quantum memory [39].

Although a qubit can exist in an a coherent superposition state, measuring the quantum state will collaps it to one of its eigenstates, $|0\rangle$ or $|1\rangle$, according to the measurement postulate of quantum mechanics i.e. the measurement will force the quantum state to lose its quantum character and changed it to just classical bits [15, 39].

The coherent superposition of exponential number of eigen states, requires also an exponential number of classical processing units to process the infomation encoed in the quantum states. The ability of quantum computer to deal with an exponential number of eigen states coherently is called the parallelism. It is uniqly a trait of quantum computer and is one of the main distiction between the quantum computer and the classical counterparts [15].
The ability of quantum state to live in coherent superposition many eigen states will subject it to quantum interference which is very important in quantum computing and facilitates the quantum operation. This phenomena has also no couterparts. Consequntly, the power of the quantum computer is exponentially larger than their classical computer [15].

1.7.1 Quantum gates

In irreversible classical computing, irreversible Boolean fonction are used whil reversible Boolean fonction are used in reversible classical computing. In quantum computing, unitary gates are used in computation [15]. Poincare (Bloch) Sphere representation provides a good idea about the possible transformations of a single qubit, since any transformation of points on the Poincare sphere is equivalent to a spesific rotation of that points by a particular angle around a particular axis [15]. Consequently any single qubit transformation is equivallent to the pruduct of the product of of two rotations around the coordinate system axes. These rotations are Rx(θ), Ry(θ), and Rz(θ) corresponding to the rotation with an angle θ around the axis x,y and z respectively [1]. Quantum gates are formed by the time evolution of the state of the quantum system described by the time dependent Schrodinger equation [15] ,

$$i\hbar\frac{d}{dt}\psi(\hat{r},t) = \hat{H}(\hat{r},t)\psi(\hat{r},t) \tag{32}$$

Two quantum bit gates are different from single qubit gates in that Two quantum bit gates provide non linear interaction between the two input qubits.The most common two qubits gates are the controlled Z (CZ) gate, swapping gate and the commonest gate, the Controlled NOT (CNOT) gate which is of our interest. They are given below in matrix form in the computational basis as [15]:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{33}$$

$$CZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{34}$$

$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{35}$$

The aforementioned matrix representation of the operators are basis dependent.Therefore, chosing the basis is very important for the implementation of quantum gates. Computational basis (logical basis B_c) is often used to represent a 2 qubit information [15].

$$B_c = |00\rangle, |01\rangle, |10\rangle, |11\rangle \tag{36}$$

An example of a unitary operator is quantum time evolution given in Eq.(38), in which the Hamiltonian H describs the quantum system. The time evolution unitary operator of the state of the quantum system is described by the time dependent Schrodinger equation Eq.(32), where $\hat{H}$ is the Homiltonian oprater which describes energies resulted from all interactions between the fields and particles by which we can determine the state of the system in space and time by just knowing the state of the system in a certain time t_o and position r. $\hat{H}$ can discribe exactly the state of the system $\psi(r,t)$ at any time t, through Eq.32 by [26]:

$$\psi(\hat{r},t) = \hat{U}(t,t_0)\psi(\hat{r},t_0) \tag{37}$$

where [15]

$$\hat{U}(t,t_0) = e^{-\imath\frac{\hat{H}}{\hbar}}(t-t_0) \tag{38}$$

so the state $|\psi(t)\rangle$ evolves through Eq.(13) [26]:

$$\psi(\hat{t}) = \sum e^{-\imath\lambda_j t}|\lambda_j\rangle\langle\lambda_j||\psi(0)\rangle \tag{39}$$

where λ_i,$|\lambda_i\rangle$ are eigen values and eigen vectors of the Hermitian operator $\hat{H}$. The unitary operator $\hat{U}$ maps the whole states in the Hilbert space to the corresponding states in the same space, which is actually rotation operation since $e^{i\hat{H}t}$ is only a phase operator , as long as H is Hermitian and all eigenvalues are real and the eigenvectors form a complete set of orthogonal vectors i.e [26].

$$|e^{i\lambda_j}| = 1 \tag{40}$$

$$\sum |\lambda_j\rangle\langle\lambda_j| = I \tag{41}$$

Since $\hat{H}$ is a squar matrix operator, therefore, it can be diagonalized and written as a sum of the form [51] :

$$\hat{H} = \sum \lambda_j |\lambda_j\rangle\langle\lambda_j| \tag{42}$$

And, since a unitary operator can be written as a function (an exponential) of a Hermitan operator [26]

$$\hat{U} = f(\hat{H}) = e^{i\hat{H}t} \tag{43}$$

So from the Silvestor theorem[26]

$$f(\hat{H}) = \sum f(\lambda_i)|\lambda_i\rangle\langle\lambda_i| \tag{44}$$

Since $\hat{H}$ is Hermitian operator, so $e^{i\hat{H}}$ is also Hermitian operator and have the same eigenvectors λ_i. Consequently, we can use a common basis for both $\hat{H}$ and $e^{i\hat{H}}$ as it was shown from Eq.(44).

The unitary transformation given in Eq.(38) illustrates that the time-evolution operator can be achieved with different Hamiltonians, since the unitary operator is only concerned with the input and output of the transformation and not depends on the intermediate state of the system. This iterprets that why there are different approaches to simulate a quantum computation in different physical systems, using photons,electrons, atoms, and other quantum systems [47].

Since the unitary operator U has the property [15]:

$$U^{-1} = U^{\dagger} \tag{45}$$

therefore the quantum computation is a physically reversible process, it is also logically reversible process. For example, if a three-qubit function mapped $|100\rangle$ to $|011\rangle$, the inverse would map $|011\rangle$ to $|100\rangle$

1.8 Universality of CNOT and 1-qubit gates

The universality is defined as capability of a certain gates to perform any unitary transformation on a multiqubit register. Single-qubit rotational operations and the CNOT two-qubit logic gates are cosidered to be sufficient for suh tasks, i.e. which makes CNOT gates this great importance [38]. In reversible classical computation, the three bitsToffoli (or Fredkin) gate is universal logic gate where, any three qubit controlled U operation (including the quantum Toffoli gate) can be decomposed into a sequence of two qubit operations, which in turn can be implemented using the single qubit rotations and CNOT gate . This interprets, why quntum CNOT gates are universal gates [15].

The universality of CNOT gates was given in the works of Cybenko et al [48] who gave a quantum circuit for any givens rotation that contains only one-qubit unitary gates and the controlled-NOT gates. Also, the fact that one-qubit gates together with the controlled-NOT gate are universal set for quantum computing was proved by Barenco et al in 1995 [49, 50] and [51].

1.9 Coherent Photon Conversion Process

In 2011 Langford et al. proposed an effective deterministic process based on Kerr-nonlinearity called Coherent Photon Conversion (CPC). The key principle of this process is the use of a strong coherent state as a classical field which pumps a third order nonlinearity and thus induces coherent oscillation between different multi-excitation states [52].

The quantum treatment of these nonlinear processes can be simplified using the concept of parametric approximation, which state that if one of coupled mode fields is highly excited coherent state, it can be assumed, at least for a certain period of time, to act like a classical field so the corresponding operator in the Hamiltonian interaction formula can be replaced by the value of the electric field. This assumption will simplify the solution of the system since it will reduce the order of nonlinearity by one [53]. The CPC process utilizes this approximation in a third order nonlinear interaction of four light fields to tune and increase the strength of coupling. CPC is used here as many other nonlinear interaction(NL) processes to create nonclassical states that are usually used in quantum information processing [52].

In the four wave mixing FWM interaction, two modes are pumped by a strong coherent state which is supplied by bright laser beam source [54, 55]. The trait of CPC is that, only one of the four interaction modes is fed with strong coherent state to pump a third order nonlinear interaction in order to create the signal and the idler fields. Therefore, it is treated classically according to parametric approximation. The coupling strength of the non linearity is thus tuned and controlled by the value of the pumped field [52].

CPC is a nonlinear optical process involving four fields interacted coherently inside a nonlinear medium which effectively considered as four-wave mixing interaction FWM created by a third-order non-linearity χ^3 [52].

The fully quantized Hamiltonian formulas composed of products of creation and annihilation operators are basic ingredients of quantum field theory and quantum optics (see e.g. [53]). In what follows, we consider the three mode Fock states $|n_{\mathsf{a}},\ n_{\mathsf{b}},\ n_{\mathsf{c}}\rangle$ on three modes a, b and c, and use the same Hamiltonian of order χ^2, defined as in [52]:

$$H_1 = \gamma_1 a b^\dagger c^\dagger + \gamma_1^* a^\dagger b c. \tag{46}$$

Here, $a, a^\dagger, b, b^\dagger, c$ and $c^\dagger$ denote the annihilation and creation operators on the modes a, b and c, respectively, and γ_1 is a scalar factor, with γ_1^* its complex conjugate. By these definitions, H_1 is classified as a Hamiltonian of χ^2-nonlinearity. In principle, these specifications are sufficient as primary assumptions to describe the processes that implement our logical gate operations. But with respect to their experimental realization, we should be aware of the general difficulties to obtain sufficiently strong nonlinear χ^2 terms in conventional optical media. To address this issue, we evoke the method of four wave mixing as described in [52] as an example of CPC. The above Hamiltonian H_1 can be realized within the coherent four wave mixing process corresponding to the Hamiltonian H_0 of χ^3 nonlinear order, given by [52]

$$H_0 = \gamma_0 a b^\dagger c^\dagger d + \bar{\gamma}_0 a^\dagger b c d^\dagger, \tag{47}$$

and by reducing this order to χ^2 via the classical field. Specifically, the scalar factor γ_1 results from the fourth photonic mode d which is "pumped" with an intense photon beam and, therefore, functions as a classical scalar factor [53]. An important property of the CPC mechanism is the possibility to tune γ_1 in H_1 with this classical beam via its electric field E, and to link

E with the coefficients γ_0 and γ_1 by [52]:

$$\gamma_1 = E\kappa\gamma_0, \tag{48}$$

where κ is a proportionality constant which depends only on the specific system features, such as the optical media and the geometric setup. The distinction between H_0 and H_1 parallels the aforementioned separation of the functional levels; while H_0 takes care of the the transition from χ^3 to χ^2 and tuning of the parameters, H_1 operates on the states encoding the quantum information [52].

It is seen from Eq.(46) that the non-linear aspects lie in the coupling between the photonic number states. But, as in any quantum system, the time evolution $\hat{U}_t\Psi$ of a state vector Ψ with $U_t = \exp(\frac{i}{\hbar}t\hat{H}_1)$ is a linear process, i.e. $\hat{U}_t$ is a linear operator on the state Hilbert space of the system. This time evolution is used to realize computational operations with deterministic performance [52].

1.10 Litrature survey

Many probabilistic and deterministic schemes have been proposed to implement photonic CNOT quantum gates. Among these schemes is the scheme introduced by Knill et al [56] which uses only linear optics elements to implement the so called linear optical quantum computing (LOQC). LOQC schemes exploit the inherent nonlinearity of photo detection process and the quantum interference phenomena to induce the nonlinearity that is required in the nondeterministic regime[58] The most important factor in the LOQC scheme is the probability of success. To satisfy the Defincenzo criteria [57] of scalable quantum computing, many schemes [59-63]. have been proposed to increase this factor, yet it is not practical to use the (LOQC) scheme for scalable quantum computing since it consumes substantial ancillary photon resources for achieving a high efficiency. The addition of ancilla appears clearly in the linear optical regime in the work of Knill etal [56] where a large number of ancilli of indistinguishable photons are introduced as a tensor product to the original computational basis based information. This addition of ancilli photons will increase the degree of freedom of the transition from one state to another and so increase the probability of success of the gate to $\frac{n}{n+1}$ where n is the number of ancilli photons [56].

This is the main obstacle to large-scale quantum information proccessing (QIP) with linear optics. On the other hand one way quantum computing [64, 65] utilizes measurement process on the probabilistic preparation of special entangled states to achieve the required non-linearity. Alternatively, a deterministic quantum computing scheme utilizes the *intrinsic* nonlinearity to realize multi-photon interactions. These deterministic quantum computing schemes are classified according to the type of nonlinearity used. Zeno gates [69-72] , use two photon absorption process as a continuous measurement to implement the nonlinearity required. Cross-Kerr coupling nonlinearity schemes [70] were proposed to implement two qubit CNOT gates, but until now there are no real materials existing which provide the sufficient induced phase shift required for quantum computing. Parity gates [71-74] are another type of nonlinear optical gate which use weak non-linearity to induce multi photon nonlinear interaction inside a Kerr media and to carry the quantum correlation from one photon to the other. To execute such interaction X- quadrature measurement are used to project the two photons states into either one of the parity subspaces and this requires reliable peak separation for discrimination the parity.

Numerous non-linear photonic devices have been investigated in theory and experiment . In this thesis we introduce a deterministic method to implement photonic quantum CNOT gates based on the CPC process.

1.11 Aim of the work

In this thesis, the goal is to build reliable mathematical models of a two qubits CNOT gates by building quantum gates in different bases and then investegating the possibility of implementing these models in reliazable physical systems.

2 Ch.2: Novel CNOT Gate Mathematical Schemes and Proposed Physical Implementation

2.1 Introduction

In this chapter two novel schemes of CNOT gate are introduced based on the bases and the type of Hamiltonian system used. Tools that are to be used in designing the CNOT gate are illustrated. It is divided into two different schemes will be presented: The first scheme includes mathematical CNOT gate architcture based on suggested Hamiltonian and on proposed three-qubits basis with an overview of the principles of the basis and the effect of the chosen basis on the final architecture of the CNOT gate design. The second scheme, includes mathematical CNOT gate scheme based on a suggested Hamiltonian and on proposed five qubit basis.

2.2 Designing tools

The standard form of CNOT gate in the computation basis is given by Eq.(33). In Dirac notation CNOT operator takes the following form in the computational basis:

$$CNOT = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 11| + |11\rangle\langle 10| \tag{49}$$

Matrix representation of operators is basis dependent while the operator representation is not. Sometimes, working in high dimensional space is useful for the reduction of the circuit implementation complexities. Related to the CNOT gate, the 2 qubits have to be nonlinearly interacted in the computational basis forming a CNOT function as in Fig.(3).

The interaction is not fixed by a certain design of CNOT gate, and the structure of the CNOT gate is governed by the basis used. Therefore, there is a wide rang of CNOT simulations.

The aforementioned matrix representation of the operator given in Eq.33

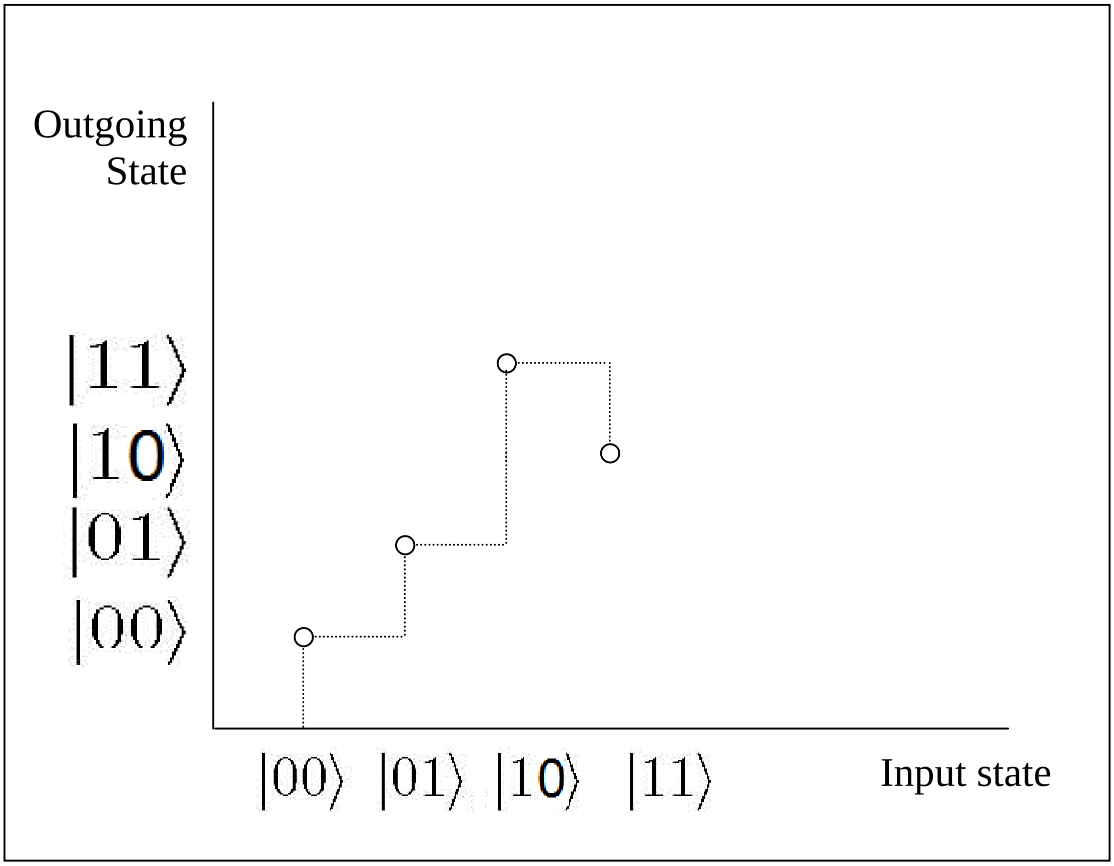

Figure 3: **The nonlinear behavior of the CNOT function in the computational basis.**

is basis dependent.Therefore, chosing the basis is very important for the implementation of quantum gates. Computational basis (logical basis B_c) given by Eq.(36) is often used to encode a 2 qubit information .

In B_c, the maximum number of possible reversible operations of any two qubit unitary function is four for the most common 2 qubit gates as shown below. Accordingly the degree of freedom in transition from any initial state to the final state is limited and fixed by one stage. To have a high degree of freedom in the transitions from initial state to the final state, more than four dimensional subspace is required by adding ancillary qubits to the computational basis B_c.

$$\begin{aligned}
|00\rangle \xrightarrow{CNOTtr_1} |00\rangle \\
|01\rangle \xrightarrow{CNOTtr_2} |01\rangle \\
|10\rangle \xrightarrow{CNOTtr_3} |11\rangle \\
|11\rangle \xrightarrow{CNOTtr_4} |10\rangle
\end{aligned}$$

$$\begin{aligned}
|00\rangle \xrightarrow{SWAPtr_1} |00\rangle \\
|01\rangle \xrightarrow{SWAPtr_2} |10\rangle \\
|10\rangle \xrightarrow{SWAPtr_3} |01\rangle \\
|11\rangle \xrightarrow{SWAPtr_4} |11\rangle
\end{aligned} \tag{50}$$

$$\begin{aligned}
|00\rangle \xrightarrow{CZtr_1} |00\rangle \\
|01\rangle \xrightarrow{CZtr_2} |01\rangle \\
|10\rangle \xrightarrow{CZtr_3} |10\rangle \\
|11\rangle \xrightarrow{CZtr_4} -|11\rangle
\end{aligned}$$

2.2.1 Three and five qubit subspaces

Adding ancilla to the computational basis according to tensor product postulate [15] will increase the flexibility of designing quantum gates to perform a gate function. For example, adding one ancilla qubit to the computational basis, will extend the dimension of the Hilbert space to eight (B_8) as suggested below:

$$B_8 : |000\rangle, |001\rangle, |010\rangle, |011\rangle|100\rangle, |101\rangle, |110\rangle, |111\rangle \tag{51}$$

Among this space we choose the subspaces B_3 such that:

$$B_3 \equiv \{|000\rangle, |001\rangle, |010\rangle, |011\rangle|100\rangle\} \tag{52}$$

Also, adding three qubits to the computational basis will extend the dimension of the space to 32, and we sellect the subspace B_5 such that:

$$B_5 \equiv \{|01010\rangle, |01001\rangle, |00110\rangle, |00101\rangle|10000\rangle\} \tag{53}$$

for the 5 qubit 5 dimensional subspace corresponding to B_3.

The redundant states are helpful because they allow to have more transitions, which means more time unitary evolution gates can be added contrary to the four dimensioned Hilbert space where the two qubit operation exhusts all the possible transitions necessary in one operation as shown in Eq.50 . The flexibility of designing a 2 qubit gate is extended and the redundant states can be used as an intermediate states to perform the final gate. In our design we add a vacuum state $|0\rangle$ to map B_c in three qubit state to extend the subspace and we suggest the possible transitions for the three qubits subspace B_3.

$$\begin{aligned}
|000\rangle \xrightarrow{op_1} |000\rangle \xrightarrow{op_2} |000\rangle \xrightarrow{op_3} |000\rangle \\
|000\rangle \xrightarrow{op_1} |001\rangle \xrightarrow{op_2} |001\rangle \xrightarrow{op_3} |001\rangle \\
|010\rangle \xrightarrow{op_1} |010\rangle \xrightarrow{op_2} |100\rangle \xrightarrow{op_3} |011\rangle \\
|011\rangle \xrightarrow{op_1} |100\rangle \xrightarrow{op_2} |010\rangle \xrightarrow{op_3} |010\rangle
\end{aligned} \tag{54}$$

In the above process given by Eq.(54) we have 3 operations,(op_1, op_2, op_3), Eq.(54) maps the 3-qubits encoded computational basis $|0\rangle|B_c\rangle \equiv |000\rangle, |001\rangle$,$|010\rangle, |011\rangle$ in a CNOT operation. Realizing or implementing a Hamiltonian system operator in the working subspace $\{B_3\}$ does realize the corresponding unitary gate in the computational basis.

2.2.2 Proposed Hamiltonian

The Hamiltonian $\hat{H}$ is a Hermitian operator corresponding to the total energy E and in classical mechanics $\hat{H}$ does always associate $\langle H\rangle$ with the expectation value of the energy of the system. This Hamiltonian is used to find the form of the time evolution of the state of the quantum system which is described by the time dependent Schrodinger equation of Eq.(32). Where $\hat{H}$ is the Homiltonian operater which describes energies resulted from all interactions between the fields and particles by which one can determine the state of the system in space and time by just knowing the state of the system in a certain time t_o and position r. $\hat{H}$ can discribe exactly the state of the system $\psi(r,t)$ at any time t, through Eq.(38).

Here a system of Hamiltonian $\hat{H}$ was suggested such that

$$\begin{aligned}
\hat{H}|000\rangle &= 0 \\
\hat{H}|001\rangle &= 0 \\
\hat{H}|010\rangle &= 0 \\
\hat{H}|011\rangle &= \gamma|100\rangle \\
\hat{H}|100\rangle &= \gamma^{\star}|011\rangle
\end{aligned} \tag{55}$$

where $|100\rangle$ is an intermediate state and never included in the input or outgoing states.

By applying quantum mechanical postulates, we can predict the out going state in this Hamiltonian interaction Eq.(55). The time evolution of quantum states in the Hamiltonian system derived from Schrodinger equation Eq.(32) is

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}t}|\psi(0)\rangle \tag{56}$$

Since the operator $e^{-\frac{i}{\hbar}\hat{\hat{H}}t}$ satisfies the relation

$$|e^{-\frac{i}{\hbar}\hat{H}t}| = I \tag{57}$$

therefore $e^{-\frac{i}{\hbar}\hat{H}t}$ is unitary operator.

From Eq.(55) the Hamiltonian matrices H_{000}, H_{001} and H_{010} corresponding to the one dimentional subspaces $\{|000\rangle\}$, $\{|001\rangle\}$ and $\{|010\rangle\}$ respectively are given as:

$$\begin{aligned} H_{000} &= (0) \\ H_{001} &= (0) \\ H_{010} &= (0) \end{aligned} \tag{58}$$

the eigen values of the above one by one matrices is calculated by the characteristic equation

$$\lambda - 0 = 0 \tag{59}$$

consequently the eigen values of the Eq.(58) are all zero. and so the time evolution of the states $|001\rangle$, $|010\rangle$,$|010\rangle$ is

$$|\psi(t)\rangle = |\psi(0)\rangle \tag{60}$$

Alternatively in the two dimensioal Hilbert space $(|011\rangle, |100\rangle)$ the interaction Hamiltonian matrix is formed from Eq.(55) as:

$$H_{011} = \begin{pmatrix} 0 & \gamma \\ \gamma^{\star} & 0 \end{pmatrix} \tag{61}$$

The eigenvalues of the matrix H_{011} of Eqs.(61) can be obtained from the characteristic equation:

$$\lambda^2 - |\gamma|^2 = 0 \tag{62}$$

$$\lambda = \pm|\gamma| \tag{63}$$

Since H_{011} is Hirmetian matrix so the eigenvectors$| \pm \lambda\rangle$ corresponding to the eigen values $\lambda = \pm|\gamma|$ can be derived by substituting Eq.(61) in the eigenvalue equations given by [75]:

$$H_{011}|+\lambda\rangle = +|\gamma||+\lambda\rangle \tag{64}$$

$$H_{011}|-\lambda\rangle = -|\gamma||-\lambda\rangle \tag{65}$$

and so the eigenvectors $|\pm\lambda\rangle$ are:

$$\begin{aligned} |\lambda_+\rangle &= \begin{pmatrix} 1 \\ \frac{\gamma^*}{|\gamma|} \end{pmatrix} \\ |\lambda_-\rangle &= \begin{pmatrix} 1 \\ -\frac{\gamma^*}{|\gamma|} \end{pmatrix} \end{aligned} \tag{66}$$

2.2.3 Time evolution unitary operator

To determine the equivalent time evolution operator of the Hamiltonian $\hat{H}$ suggested by Eq.(55). The Hamiltonian matrix has to be transformed to its equivallent diagonal matrix form $\hat{H}_D$ according to the equivallence princible which states that two matrices are similar if they have the same characteristic equation [75].

The diagonal form of an operator matrix is more easier to deal with function of operator matrix than that with the original matrix since it can be directly execute the function by applying the function on each diagonal element. The matrix $\hat{H}$ of Eq.(61) is Hermitian and has the diagonal form [75]

$$D = S^{-1}HS = \begin{pmatrix} d_{11} & 0 \\ 0 & d_{22} \end{pmatrix} \tag{67}$$

where S is the modal matix whose columns are the eigenvectors of$\hat{H}$ i.e [75]

$$S = \{|\lambda_+\rangle, |\lambda_-\rangle\} = \begin{pmatrix} 1 & 1 \\ \frac{\gamma^*}{|\gamma|} & -\frac{\gamma^*}{|\gamma|} \end{pmatrix} \tag{68}$$

Therefore, from Eq.(67), the diagonal form of the matrix $\hat{H}$ is:

$$D = \begin{pmatrix} |\gamma| & 0 \\ 0 & -|\gamma| \end{pmatrix} \tag{69}$$

The time evolution unitary matrix $f(H)$ can be determined by:

$$f(H) = S \begin{pmatrix} f(d_{11)}) & 0 \\ 0 & f(d_{22}) \end{pmatrix} S^{-1} \tag{70}$$

Calculating $f(H) = e^{i\frac{H}{\hbar}t}$ in the specified basis helps to find the time evolution operator $\hat{U}$ of Schrodinger's equation Eq.(32). But since [75]

$$f(\hat{H}) = Sf(D)S^{-1} = Se^{i\frac{D}{\hbar}}S^{-1} \tag{71}$$

where $f(D)$ is the function operation on the equivallence diagonal form D given in Eq.(69) of the Hamiltonian matrix $\hat{H}$ in the specified subspace as:

$$f(D) = \begin{pmatrix} f(d_{11}) & 0 \\ 0 & f(d_{22}) \end{pmatrix} \tag{72}$$

from Eqs.(67,68,70, 72), the following equation will be found:

$$f(\hat{H}) = \begin{pmatrix} \frac{1}{2}e^{i\frac{|\gamma|}{\hbar}t} + \frac{1}{2}e^{-i\frac{|\gamma|}{\hbar}t} & \frac{1}{2}\frac{\gamma^*}{|\gamma|}(e^{i\frac{|\gamma|}{\hbar}t} - \frac{1}{2}e^{-i\frac{|\gamma|}{\hbar}t}) \\ \frac{1}{2}\frac{\gamma^*}{|\gamma|}(e^{i\frac{|\gamma|}{\hbar}t} - \frac{1}{2}e^{-i\frac{|\gamma|}{\hbar}t}) & \frac{1}{2}e^{i\frac{|\gamma|}{\hbar}t} + \frac{1}{2}e^{-i\frac{|\gamma|}{\hbar}t} \end{pmatrix} \tag{73}$$

The outgoing states of the unitary time evolution operator of Eq.(73,60) of the suggested Hamiltonian given by Eq.(55) are as follows:

$$
\begin{aligned}
|000\rangle &\xrightarrow{\hat{U}_t} |000\rangle \\
|001\rangle &\xrightarrow{\hat{U}_t} |001\rangle \\
|010\rangle &\xrightarrow{\hat{U}_t} |010\rangle \\
|011\rangle &\xrightarrow{\hat{U}_t} \cos(\tfrac{|\gamma|}{\hbar}t)|011\rangle + i\sin(\tfrac{|\gamma|}{\hbar}t)|100\rangle \\
|100\rangle &\xrightarrow{\hat{U}_t} \cos(\tfrac{|\gamma|}{\hbar}t)|100\rangle + i\sin(\tfrac{|\gamma|}{\hbar}t)|011\rangle
\end{aligned}
\tag{74}
$$

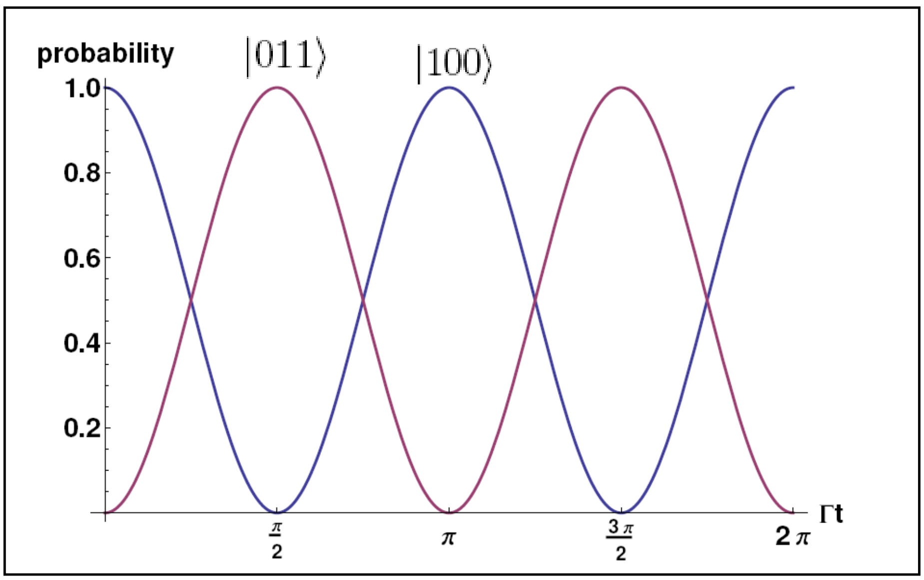

Figure 4: **Timing evolution equation given by Eq.(74) starting from the input state$|100\rangle$. It shows the resulting Rabi-like oscillations [55] for the probabilities amplitude of the two basis states $|100\rangle$ and $|011\rangle$ as a function of the interaction parameter $\Gamma t = \frac{|\gamma|}{\hbar}t$.The coherent oscillation of the superposition state composed of the state $|100\rangle$ associated with cosine curve and $|011\rangle$ associated with sine curve is very clear and for a certain value the interaction parameter Γt we can pick up the oscillation at the desired state.**

Starting from the input state $|100\rangle$, Fig.(4) shows the resulting Rabi- like oscillations [52] for the probability amplitudes of the two eigenstates , $|100\rangle$ with bold (cosine) curve and $|011\rangle$ with light (sine) curve as a function of the interaction parameter$\frac{|\gamma|}{\hbar}t$. i.e every eigenstate in the subspace $\{|100\rangle$, $|011\rangle\}$ evolved to a coherent superposition state. For a certain time t that makes the interaction parameter equal π , the evolution matrix takes the matrix form given in Eq.(34) and performs the controlled Z operator in the Fock basis (number state basis), which is exactly the result reached by Langford et al [52].

Since the number state basis is used here, therefore, using controlled Z operator to develop another two-qubit quantum gate, such as CNOT gate, requires designing all other single qubit gates in the number state basis which is another obstacles in the 2 qubit gate designing plans. Also, to avoid the walk-off condition one needs to deal with shorter length of nonlinear material [54]. Instead, we follow another approaches in order to construct CNOT gate from the elementary process. The four-extra states in the 3 qubit, 8 dimensional Hillbert space may be used to extend the horizon of CNOT gate implementation such that imagined by Eq.(54).

The time evolution equation given in Eq.(74) in the 3 qubits basis B_8 can be acheived in the 5 qubit subspace B_5 given in the next section by suggesting the same Hamiltonian matrices elements, H_{000} in the subspace $\{|01010\rangle\}$, H_{001} in the subspace $\{|01001\rangle\}$ and H_{010} in the subspace $\{|00110\rangle\}$ and that given in Eq.(61) for the subspace denoted by $\{|00101\rangle, |10000\rangle\}$.

2.3 Three qubit basis CNOT gate architecture

Here we use some of these extra states ,e.g., $|100\rangle$ to construct other quantum reversible gates $(U_1, U_2, ...U_n)$ working in the subspace B_3 aimed to realize CNOT gate from the combination of these gates such that:

$$CNOT = \hat{U}_1 \hat{U}_2.....\hat{U}_n \tag{75}$$

In plain, we control the interaction between the qubits for a certain and countable time interval t for many processes and then match all these processes to perform a CNOT gate.

Now, let the evolution time for $\hat{U}_1$ in Eq.(74) equal t such that the time interaction parameter in Fig.(4) equal $\frac{\pi}{2}$, then:

$$\begin{aligned}
&|abc\rangle \\
&|000\rangle \xrightarrow{\hat{U}_{\frac{\pi}{2}}} |000\rangle \\
&|001\rangle \xrightarrow{\hat{U}_{\frac{\pi}{2}}} |001\rangle \\
&|010\rangle \xrightarrow{\hat{U}_{\frac{\pi}{2}}} |010\rangle \\
&|011\rangle \xrightarrow{\hat{U}_{\frac{\pi}{2}}} \frac{\gamma}{|\gamma|}|100\rangle
\end{aligned} \tag{76}$$

exchanging the qubits a and b the outgoing of Eq.76 results in

$$\begin{aligned}
&|000\rangle \xrightarrow{U_{swap}} |000\rangle \\
&|001\rangle \xrightarrow{U_{swap}} |001\rangle \\
&|010\rangle \xrightarrow{U_{swap}} |100\rangle \\
&\frac{\gamma}{|\gamma|}|100\rangle \xrightarrow{U_{swap}} \frac{\gamma}{|\gamma|}|010\rangle
\end{aligned} \tag{77}$$

To return to the computational basis another $U_{\pi/2}$ section is required such that

$$\begin{aligned}
&|000\rangle \xrightarrow{U_{\frac{\pi}{2}}} |000\rangle \\
&|001\rangle \xrightarrow{U_{\frac{\pi}{2}}} |001\rangle \\
&|100\rangle \xrightarrow{U_{\frac{\pi}{2}}} \frac{\gamma^*}{|\gamma|}|011\rangle \\
&\frac{\gamma}{|\gamma|}|010\rangle \xrightarrow{U_{\frac{\pi}{2}}} \frac{\gamma}{|\gamma|}|010\rangle
\end{aligned} \tag{78}$$

The over all transition matrix constructed from transitons of Eqs.(76-78) can be written in the computational basis as:

$$U_T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\gamma^*}{|\gamma|} \\ 0 & 0 & \frac{\gamma}{|\gamma|} & 0 \end{pmatrix} \tag{79}$$

Eq.(79) is similar to the standard CNOT gate given by Eq.(33) if we let the entry γ to be real unity and then correct the phase factor i by applying a single phase gate of $\frac{\pi}{2}$ phase shift to the qubit b given by:

$$\phi_{\pi/2} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix} \tag{80}$$

The standard CNOT matrix in the basis B_3 is simulated by the sequential operations of Eqs.(76),(77), and (78) which can be abbreviated by (USU).

2.4 Five qubit basis CNOT gate architecture

As seen before, the dimension of basis will affect the method of designing quantum gates by extending the choices of designing intermediate gates. This facility can be utilized to design another architecture of a CNOT gate in different basis. Here, we suggest a five dimensional subspace B_5 including the states:

$$B_5 \equiv \{|01010\rangle, |01001\rangle, |00110\rangle, |00101\rangle, |10000\rangle\} \tag{81}$$

We also propose the type of the Hamitonian interaction in the subspace B_5 to be

$$\begin{aligned}
H|01010\rangle &= 0 \\
H|01001\rangle &= 0 \\
H|00110\rangle &= 0 \\
H|00101\rangle &= i\alpha|10000\rangle \\
H|10000\rangle &= i\alpha^\star|00101\rangle
\end{aligned} \tag{82}$$

where α is a complex number interaction coefficient. The time evolution matrix for this Hamiltonian interaction given by Eq.(82) in the sub spaces $\{|01\dot{0}10\rangle\}$, $\{|01\dot{0}01\rangle\}$, $\{|00\dot{1}10\rangle\}$ is 1×1 identity matrices , while the time evolution matrix for the subspace $\{|00101\rangle\dot{,}|10000\rangle\}$ is identical to that of Eq.(74).

To see the time evolution unitary operator of the states $|abcde\rangle$ of B_5 under interaction parameter $\pi/2$ (where $a, b, c, d,$ and e are the qubits of the 5 qubits state), the interaction will transform the states in the subspace B_{51} to B_{52} as :

$$\begin{array}{ccccc} \multicolumn{5}{c}{B_{51}} \\ \hline a & b & c & d & e \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{array} \xrightarrow{U_{\frac{\pi}{2}}} \begin{array}{ccccc} \multicolumn{5}{c}{B_{52}} \\ \hline & & & & \\ i\frac{\alpha}{|\alpha|} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{array} \tag{83}$$

To allow the evolution of other states e.g; $|00110\rangle$, swapping in qubits d and e is convenient for interaction. The over all transitions from the subspace

B_{52} to the subspace B_{53} is governd by as:

$$\begin{array}{ccccc} & & B_{52} & & \\ \hline a & b & c & d & e \\ i\frac{\alpha}{|\alpha|} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{array} \xrightarrow{5swap1} \begin{array}{ccccc} & & B_{53} & & \\ \hline & & & & \\ i\frac{\alpha}{|\alpha|} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array} \tag{84}$$

Now, we can suggest another evolution operation in the subspace B_{53} with $\pi/2$ interaction parameter which transits the states to the subspace B_{54} as:

$$\begin{array}{ccccc} & & B_{53} & & \\ \hline a & b & c & d & e \\ i\frac{\alpha}{|\alpha|} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array} \xrightarrow{U_{\frac{\pi}{2}}} \begin{array}{ccccc} & & B_{54} & & \\ \hline & & & & \\ 0 & 0 & -1 & 0 & 1 \\ i\frac{\alpha^*}{|\alpha|} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array} \tag{85}$$

To return to the original basis B_{51} we have to do successive swapping and evolution operation of the form:

$$\begin{array}{ccccc} & & B_{54} & & \\ \hline a & b & c & d & e \\ 0 & 0 & -1 & 0 & 1 \\ i\frac{\alpha^*}{|\alpha|} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array} \xrightarrow{5swap3} \begin{array}{ccccc} & & B_{55} & & \\ \hline & & & & \\ 0 & 0 & -1 & 0 & 1 \\ i\frac{\alpha^*}{|\alpha|} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{array} \tag{86}$$

and

$$
\begin{array}{ccccc c ccccc}
\multicolumn{5}{c}{B_{55}} & & \multicolumn{5}{c}{B_{56}} \\
\hline
a & b & c & d & e & & & & & & \\
0 & 0 & -1 & 0 & 1 & \overset{U_{\frac{\pi}{2}}}{\longrightarrow} & 0 & 0 & -1 & 1 & 0 \\
i\alpha^{\star} & 0 & 0 & 0 & 0 & & 0 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & & 0 & 1 & 0 & 1 & 0
\end{array}
\tag{87}
$$

The over all sequential operations of Eqs.(83),84,(85),(86),and (87), abbreviated by $USUSU$ perform a standard CNOT gate after a phase correction with a π single qubit phase gate on qubit c.

2.5 Proposed physical realization with coherent photon conversion (CPC)

In this section, we implement the mathematical entities used in designing the CNOT gate that was derived in this chapter into physical entities. The next subsection is allocated to implement the suggested basis B_3 and B_5 and the proposed Hamiltonians given in Eqs.(55, 82). In the third subsection we talk about the state of the art tools in Quantum Information processing, the coherent photon conversion (CPC).

2.5.1 Basis representation

Here, we associate the three qubits basis B_3 selected previously in this chapter with the energy-time coordinate basis; of the the 3-mode number state basis $B_F \in |n_1 n_2 n_3\rangle$ where n_i here is the number of photons. Restriclly speeking we choose a bounded 3-mode Fock basis of three eigenstates such that $n_i \in \{0, 1\}$ [37] to encode the basis given in Eq.(52). The energy of number state $|n\rangle$ is:

$$E_n = \hbar\omega(n + \frac{1}{2}) \tag{88}$$

For $n = 0$, the number state is called vacuum state with energy $E_o = \frac{1}{2}\hbar\omega$.

The degree of freedom in this case is unbounded, so the dimension of the space is also unbounded . Since the evolution of the system is done by unitary operator, it dictates that we have to deal with bounded subspace for example with number states $|n\rangle$ for $n \leq 1$. In this case we use the two dimensional number state basis $|0\rangle$ and $|1\rangle$ to encode the logical state $|0\rangle, |1\rangle$ respectively, i.e. in the implementation of the 3 qubits basis, every logical qubit is implemented from the two dimensional Fock basis $\{|0\rangle, |1\rangle\}$.

Alternatively, we associate the five qubits basis, B_5, with the space momentum coordinate basis. This is performed in terms of polarization coordinates since in photonic quantum information the qubits are commonly represented by photons. The logical basis used $(|0\rangle, |1\rangle)$ for a two level system is encoded in the direction of the momentum of photons.Therefore, the vertical polarizition state $|V\rangle$ and horizontal polarization state $|H\rangle$ are considered to correspond to the logical basis $|1\rangle$ and $|0\rangle$, respectively.

2.5.2 System Hamiltonian

To implement the Hamiltonian suggested in Eq.(55) and Eq.(82), we evoke the coherent photon conversion process introduced by Langford et al [52] to implement the Hermitian operator of Eq.(55) in the extended computational basis B_8 and finally to implement the CNOT gate in both the

Fock basis and polarization basis. The distinction between H_0 and H_1 of Eq.(47,46) parallels the aforementioned separation of the functional levels; while H_0 takes care of the the transition from χ^3 to χ^2 and tuning of the parameters, H_1 operates on the states encoding the quantum information. Eq.(46) confimed that the non-linear aspects of the CPC lie in the coupling between the photonic number states. But, as in any quantum system, the time evolution $\hat{U}_t\Psi$ of a state vector Ψ with $U_t = \exp(\frac{\mathrm{i}}{\hbar}t\hat{H}_1)$ is a linear process, i.e. $\hat{U}_t$ is a linear operator on the state Hilbert space of the system which interprets, why this time evolution is used to realize computational operations with deterministic performance. Our CNOT gates operate within the 5-dimensional subspace B_3 of Eq.(52) spanned by the number state basis $\mathcal{B} = \{\Psi_j\}_{j=0}^{4}$, with $\Psi_j = |\beta_2\ \beta_1\ \beta_0\rangle$, where $\{\beta_2\ \beta_1\ \beta_0\} \in 0,1$ in binary notation. Note that H_1 in Eq.(46) can be constructed in the matrix form within the basis $\mathcal{B}$. This matrix is semillar to the suggested Hamiltonian of Eq.(55) which annihilates the state vectors $|\Psi_j\rangle$ for $j = 0,1,2$ and it exchanges the state vectors $\Psi_3 = |0\ 1\ 1\rangle$ and $\Psi_4 = |1\ 0\ 0\rangle$ up to the phase factors γ or γ^*, respectively. By the general principles of quantum mechanics, it follows that the corresponding time evolution $\hat{U}_{t,1} = \exp(\frac{\mathrm{i}}{\hbar}t\hat{H}_1)$ acts as a coherent Rabi oscillationgiven in Fig.4 between $|0\ 1\ 1\rangle$ and $|1\ 0\ 0\rangle$ within the 2D subspace they span. In summary, we can derive a unitary time evolution operator semillar to that of Eq.(73,74),

$$U_{t,1}|\Psi_j\rangle = |\Psi_j\rangle, (j = 0,1,2), \tag{89}$$

$$U_{t,1}(|0\ 1\ 1\rangle) = \cos\left(\frac{|\gamma_1|}{\hbar}t\right)|0\ 1\ 1\rangle + \mathrm{i}\frac{\gamma_1}{|\gamma_1|}\sin\left(\frac{|\gamma_1|}{\hbar}t\right)|1\ 0\ 0\rangle, \quad (j = 3) \tag{90}$$

$$U_{t,1}(|1\ 0\ 0\rangle) = \cos\left(\frac{|\gamma_1|}{\hbar}t\right)|1\ 0\ 0\rangle + \mathrm{i}\frac{\gamma_1^*}{|\gamma_1|}\sin\left(\frac{|\gamma_1|}{\hbar}t\right)|0\ 1\ 1\rangle, \quad (j = 4). \tag{91}$$

2.6 Archetecturing of the three qubit based CNOT gate with CPC

Here, we implement the 3 qubits based scheme mentioned in this chapter with the CPC process and by encoding the three qubits basis with the number state basis B_F If the selected coordinate of the photonic system is time- energy coordinate, the number (Fock) states $|n\rangle$ are used as the basis for $n \geq 0$.

In general, a quantum CNOT gate maps the computational basis $\mathcal{B}_{\mathcal{C}} = \{|C\ T\rangle\}_{C,T=0,1}$ to itself, with C denoting the control qubit, and T the target qubit of a bipartite (two photon) system. The matrix $M_{\mathcal{C}}$ of the lossless CNOT gate with respect to $\mathcal{B}_{\mathcal{C}}$, and with possible nontrivial phase factors is such that

$$\begin{array}{c} C\ T \\ \left.\begin{array}{c} |0\ 0\rangle \\ |0\ 1\rangle \\ |1\ 0\rangle \\ |1\ 1\rangle \end{array}\right\} \end{array} \xrightarrow{M_{\mathcal{C}}} \begin{array}{c} C\ T \\ \left\{\begin{array}{c} \mathrm{e}^{\mathrm{i}\phi_{11}}|0\ 0\rangle \\ \mathrm{e}^{\mathrm{i}\phi_{22}}|0\ 1\rangle \\ \mathrm{e}^{\mathrm{i}\phi_{43}}|1\ 1\rangle \\ \mathrm{e}^{\mathrm{i}\phi_{34}}|1\ 0\rangle \end{array}\right. \end{array}, \quad M_{\mathcal{C}} = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\phi_{11}} & 0 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\phi_{22}} & 0 & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}\phi_{34}} \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\phi_{43}} & 0 \end{pmatrix}. \quad (92)$$

The exponentials $\mathrm{e}^{\mathrm{i}\phi_{ij}}$ in Eq.(92) are arbitrary phase factor of the matrix elements $(M_{\mathcal{C}})_{ij}$ which satisfy $\phi_{34} = -\phi_{43}$. This structure provides the unitarity of the linear map $M_{\mathcal{C}}$, which therefore represents an ideal, lossless gate on the Hilbert space of the bipartite quantum system.

We note that in a quantum computational devices, a phase shift in any of the output states does not affect the probability of measurement of the state. Therefore, each of the entries of $M_{\mathcal{C}}$ may be multiplied by a phase factor $e^{\varphi_{kj}}$ without changing the reading process of the output. The lossless unitary matrix given in Eq.(92) can be obtained by a certain arrangements of CPC components with the CPC behaviour given in Eqs.(89-91) in the subspace $\mathcal{B}_C$ which can be used to implement the sequential operations of Eqs.(76),(77), and (78) to implememt the function of the standard CNOT gates of Eq.(33) in the number state and polarization bases as will be illustrated in the next chapter.

3 Ch.3: Implementation of quantum CNOT gates with CPC

3.1 Introduction

In this chapter, we have two CNOT gate implementation schemes. First the 3-qubits basis based CNOT gate scheme in a physical model will be constructed. Next, a 5-qubit basis based CNOT gate derived in chapter two is imlemented in the polarization basis physical model.

3.2 Number state based CNOT gate architecture

Here, three mode number state is used to encode a three qubit information. Our CNOT gate design given in Fig.(6) also uses an ancillary state A in addition to the control C and target T state, to encode the computational states $|A\ C\ T\rangle$ as the previously defined photonic number states $|\Psi_j\rangle = |\beta_2\ \beta_1\ \beta_0\rangle$, such that mode a in CPC $U_{\frac{\pi}{2}}$ carries the ancilla A, which at the input stage remains in the vacuum state $|0\rangle_a$, mode b carries the control qubit C and mode c the target qubit T. In particular, for a fixed time t of evolution, such that the interaction parameter $\Gamma t = \frac{\pi}{2}$, the transition of the input states is evolved according to Eq.(89) and Eqs.(63,83,84,85) as:

$$\left.\begin{array}{c} \scriptstyle a\ b\ c \\ |0\ 0\ 0\rangle \\ |0\ 0\ 1\rangle \\ |0\ 1\ 0\rangle \\ |0\ 1\ 1\rangle \end{array}\right\} \xrightarrow{U_{\frac{\pi}{2}}} \left\{\begin{array}{c} \scriptstyle a\ b\ c \\ |0\ 0\ 0\rangle \\ |0\ 0\ 1\rangle \\ |0\ 1\ 0\rangle \\ \frac{i\gamma}{|\gamma|}|1\ 0\ 0\rangle \end{array}\right. . \tag{93}$$

This unitary operation is the first of two CPC sections used to implement our USU CNOT gate given in Eqs.(89-91). The complete CNOT scheme is realized by two additional operations. First, we simply swap the modes a and b, and we denote this operation by S_{ab}. Finally, we apply the operation

$U_{\frac{\pi}{2}}$ once more. Labeling the states by A, C and T for ancillary, control an the target qubit fed to modes a, b and c, this amounts to the following process

$$\begin{array}{cc} j & A\,C\,T \\ 0 & |0\;0\;0\rangle \\ 1 & |0\;0\;1\rangle \\ 2 & |0\;1\;0\rangle \\ 3 & |0\;1\;1\rangle \end{array}\left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\end{array}}\right\} \xrightarrow{U_{\frac{\pi}{2}}} \left\{\begin{array}{c} |0\;0\;0\rangle \\ |0\;0\;1\rangle \\ |0\;1\;0\rangle \\ \frac{i\gamma}{|\gamma|}|1\;0\;0\rangle \end{array}\right\} \xrightarrow{S_{ab}} \left\{\begin{array}{c} |0\;0\;0\rangle \\ |0\;0\;1\rangle \\ |1\;0\;0\rangle \\ \frac{i\gamma}{|\gamma|}|0\;1\;0\rangle \end{array}\right\} \xrightarrow{U_{\frac{\pi}{2}}} \left\{\begin{array}{c} A\,C\,T \\ |0\;0\;0\rangle \\ |0\;0\;1\rangle \\ \frac{i\gamma^\star}{|\gamma|}|0\;1\;1\rangle \\ \frac{i\gamma}{|\gamma|}|0\;1\;0\rangle \end{array}\right. \tag{94}$$

Obviously, the entire operation $U_{\frac{\pi}{2}}S_{ab}U_{\frac{\pi}{2}}$ performs the CNOT operation on the controll and target qubits $|C\rangle$ and $|T\rangle$ as required, appart from the phase factors $\frac{i\gamma}{|\gamma|}$ and $\frac{i\gamma^\star}{|\gamma|}$ aquired by the pure output states $|T\;C\rangle$ equal to $|0\;1\rangle$ and $|1\;1\rangle$. If required, these phase factors can be compensated for via the tunability of γ. In particular, choosing the classical electric field component E to be $\frac{\mathbf{E_r}}{\gamma_{\mathbf{or}}} = -\frac{\mathbf{E_i}}{\gamma_{\mathbf{oi}}}$, **where $E_r, \gamma_{or}, E_i, \gamma_{oi}$ are the real and imaginary parts of E and γ_o** Eq.48 gives $\gamma = 1$. The remaining factor i in Eq.(94) can be removed by applying a single qubit phase shift of $\frac{\pi}{2}$ on the control mode C.

The swapping procedure can be done by hard swapping and may be done with linear optics. There are several ways of dealing with this compatibility issue. The CPC mechanism is sufficiently flexible to allow for variation of the input and output frequencies (colors) of the modes. Moreover, linear swapping mechanism can be applied on pairs of photonic qubits.

It is also possible to avoid swapping between different modes altogether by using two different forms of the Hamiltonian for the time evolution operators. We note that the combination $U_{\frac{\pi}{2}}S_{a,c}$ (after the first application of $U_{\frac{\pi}{2}}$) by Eq.(94) is equivalent to using the time evolution according to the Hamiltonian

$$\hat{H}_2 = \gamma\hat{a}^\dagger\hat{b}\hat{c}^\dagger + \bar{\gamma}\hat{a}\hat{b}^\dagger\hat{c}, \tag{95}$$

and subsequent relabeling of the output modes as CAT. Indexing the

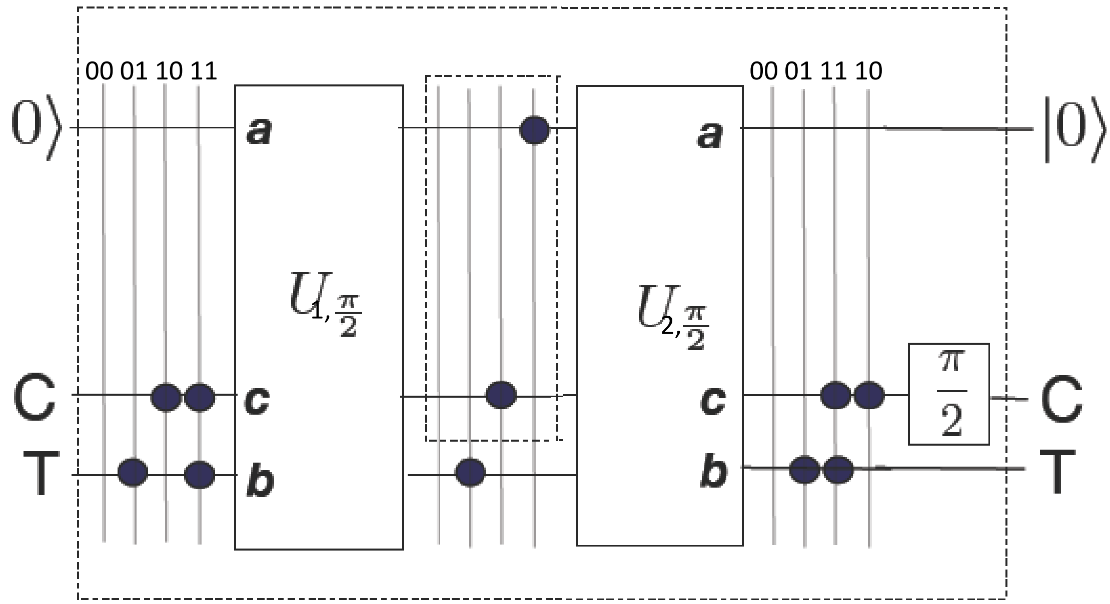

Figure 5: **Deterministic USU CNOT gate scheme in the Fock state basis B. This scheme mainly consists of 2 elements of CPC ($\hat{U}_{1,\frac{\pi}{2}}$) and ($\hat{U}_{2,\frac{\pi}{2}}$). The swapping operation between modes a and b is performed by self operatio of ($U_{1,\frac{\pi}{2}}$) and ($U_{2,\frac{\pi}{2}}$) and hence there is no need to hard swapping. The black circles represents the single photons of the number states input to CPC ($U_{\frac{\pi}{2}}$)at modes a , b and c. The block, $\frac{\pi}{2}$, represents single quantum bit phase gate of $\pi/2$ phase shift. The three qubits in the I/p state are , the vacuum state $|0\rangle$ at mode a, the control qubit C at mode c and the target qubit T at mode b**

unitary operators as in Fig.(5), we have $\hat{U}_{n,\frac{\pi}{2}} = \exp(\frac{\pi}{2}\mathrm{i}\hat{H}_n)$, $(n = 1, 2)$, and in analogy with (94) we obtain the process

$$\begin{array}{cc} j & A\,C\,T \\ 0 & |0\;0\;0\rangle \\ 1 & |0\;0\;1\rangle \\ 2 & |0\;1\;0\rangle \\ 3 & |0\;1\;1\rangle \end{array} \left.\vphantom{\begin{array}{c}a\\a\\a\\a\end{array}}\right\} \xrightarrow{U_{1,\frac{\pi}{2}}} \left\{\begin{array}{c} |0\;0\;0\rangle \\ |0\;0\;1\rangle \\ |0\;1\;0\rangle \\ \frac{\mathrm{i}\gamma}{|\gamma|}|1\;0\;0\rangle \end{array}\right\} \xrightarrow{U_{2,\frac{\pi}{2}}} \left\{\begin{array}{c} C\;AT \\ |0\;0\;0\rangle \\ |0\;0\;1\rangle \\ \frac{\mathrm{i}\gamma^\star}{|\gamma|}|1\;0\;1\rangle \\ \frac{\mathrm{i}\gamma}{|\gamma|}|1\;0\;0\rangle \end{array}\right. . \tag{96}$$

Here, there is no intermittent change between modes in the internal input and output states.

The fact that the CNOT operation $M_{\mathcal{C}} = U_{2,\frac{\pi}{2}} U_{1,\frac{\pi}{2}}$ can be realized as a product of two unitary operators with exponentials of $\hat{H}_1$ and $\hat{H}_2$ respectively, raises the question whether $M_{\mathcal{C}}$ is itself the exponential of a (possibly simple) Hamiltonian. The Baker-Campbell-Hausdorff formula [28] provides a Hermitiean operator H_{12} such that

$$\hat{U}_{2,\frac{\pi}{2}} \hat{U}_{1,\frac{\pi}{2}} = \exp\left(\frac{\pi}{2} \mathrm{i} \hat{H}_2\right) \exp\left(\frac{\pi}{2} \mathrm{i} \hat{H}_1\right) = \exp(\mathrm{i} \hat{H}_{12}). \tag{97}$$

In general, H_{12} is an infinite series in the commutators expressions of any order of H_1 and H_2. The series may become simple in case the commutators are zero from some low order onwards. But straightforward calculations reveal that in the present case, these commutator polynomials of $\hat{H}_1$ and $\hat{H}_2$ are nonzero, and even map elements of $\mathcal{B}$ to states that are orthogonal to $\mathcal{B}$.

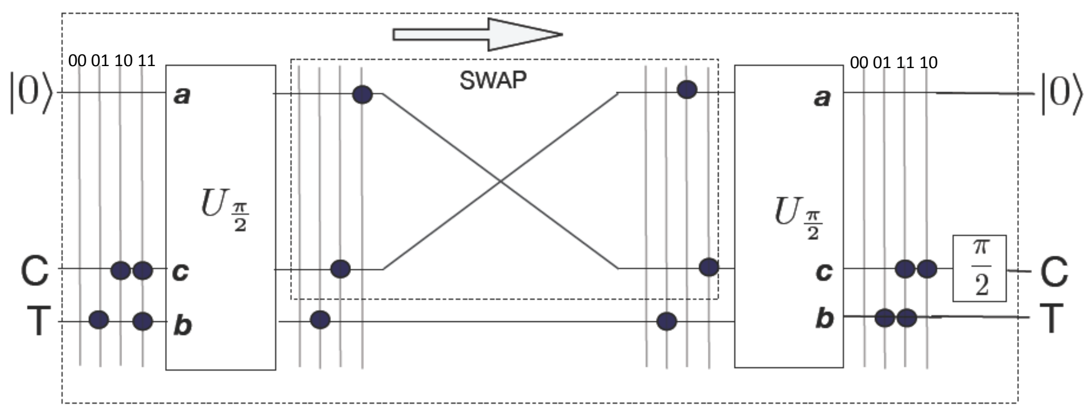

Figure 6: **Deterministic USU CNOT gate scheme in the Fock state basis $\mathcal{B}$. This scheme mainly consist of 2 elements of CPC ($U_{\frac{\pi}{2}}$) and swapping element SWAP. The swapping operation between modes a and b is performed by hard swapping or by the system of beam splitter (BS) and phase shifting (PS) elements. The black circles represents the single photons of the number states input to CPC ($U_{\frac{\pi}{2}}$)at modes a , b and c. The block, $\frac{\pi}{2}$, represents single quantum bit phase gate of $\pi/2$ phase shift. The three qubits in the I/p state are , the vacuum state $|0\rangle$ at mode a, the control qubit C at mode c and the target qubit T at mode b**

3.3 Polarization based CNOT gate architecture

The idea of our second CNOT gate construction is to encode the qubits as two photon polarization states and to process them with the CPC mechanism. The proposed scheme employs three CPC sections of the same type as previously, i.e. with three photonic modes of which one is ancillary, the other two carry specific polarization states of the qubits. Writing H for "horizontal" and V for "vertical" polarization, we obtain the basis $\mathcal{B}_p = \{|H\ H\rangle, |H\ V\rangle, |V\ H\rangle, |V\ V\rangle\}$, i.e. $\mathcal{B}_p = (|p_1, p_2\rangle)_{p_1,p_2=H,V}$. Each polarization state $|p_n\rangle$ is mapped to a bimodal state by $|H\rangle \mapsto |1\ 0\rangle$, $|V\rangle \mapsto |0\ 1\rangle$, as shown by the transformation section in Fig.7

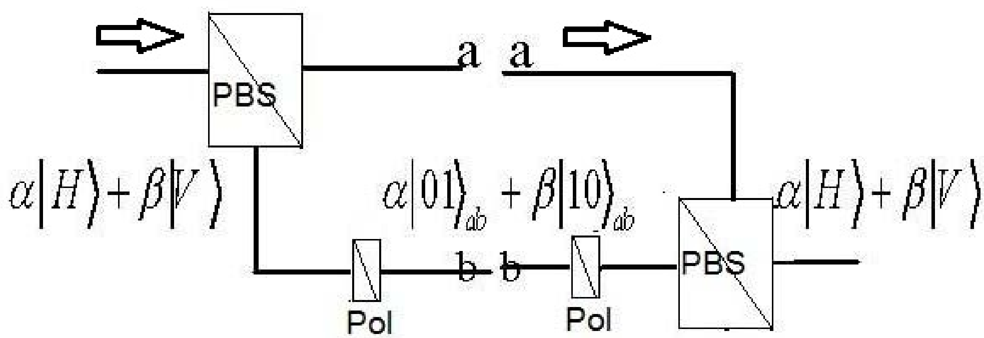

Figure 7: **Transformation from dual rail to polarization encoding and vice versa.The large blocks represent the totally polarized beam splitters PBS and the small blocks represent the polarizers POL. The superposition polarized state $\alpha|H\rangle + \beta|V\rangle$ is transformed to the two routs state $\alpha|10\rangle + \beta|01\rangle$ and vice versa.**

(for $n = 1, 2$), whereby the first (p_1) and second (p_2) components encodes the target and control state respectively, with the corresponding bimodal states being denoted $|T_1, T_2\rangle$ and $|C_1, C_2\rangle$, respectively. This extends to the product states in the obvious way, $|p_1\ p_2\rangle \mapsto |T_1\ T_2\ C_2\ C_2\rangle$, and the resulting states form the dual rail basis $\mathcal{B}_d = \{|1\ 0\ 1\ 0\rangle, |1\ 0\ 0\ 1\rangle, |0\ 1\ 1\ 0\rangle, |0\ 1\ 0\ 1\rangle\}$, which spans a 4D subspace within the Fock space of the four modes T_1, T_2, C_1, and C_2. In practice, this transition from $\mathcal{B}_p$ to $\mathcal{B}_d$ is achieved by passing the states p_1 and p_2 through the left section of the transformer of Fig.(7)

The complete dual rail CNOT gate is given the circuit diagram of Fig.(8). After splitting the input beams into their polrarizations, and setting the ancilla A to the vacuum state, the CPC mechanism is applied.

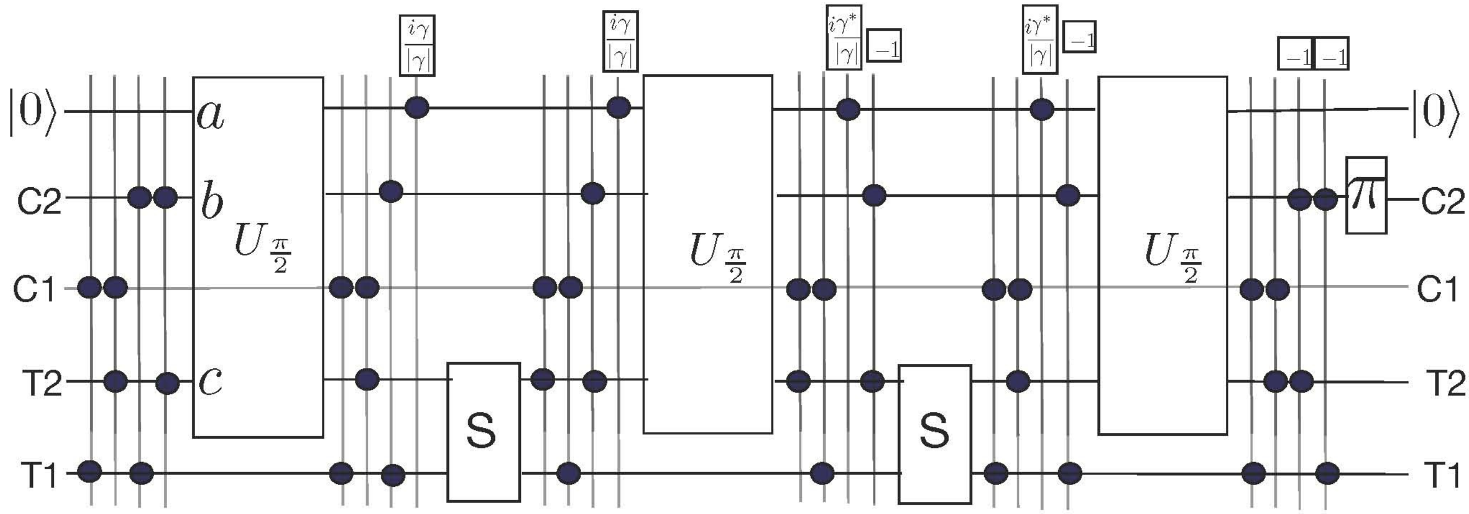

Figure 8: **The deterministic dual rail USUSU CNOT gate. The backbone of this gate involves mainly three sections of CPC ($U_{\frac{\pi}{2}}$), two swapping elements S and single qubit phase correcter π The polarization basis input state (CT) is passed through a polarization to dual rail transformer given in Fig.7 to C1C2 T1T2 and distributed as single photons on the dual rail modes denoted by a black circles. The vacuum state $|0\rangle$ is an ancilla qubit. The dual rail output can be transformed back into polarization states via dual rail to polarization transformer given in Fig.7. Note that swapping operation is excuted between the two qubits T1T2 of the dual rail basis target qubit T. The five qubits $|0\rangle$C1C2T1T2 interaction through the components $U_{\frac{\pi}{2}}$, S, $U_{\frac{\pi}{2}}$, S, $U_{\frac{\pi}{2}}$ performs exactly a CNOT function.**

The qubits T_1 and C_1 are fed into the modes b and c, respectively, to perform the unitary transform $U_{\frac{\pi}{2}}$ as defined previously in Eq.(97), and the output at c is then swapped with qubit C_2 (swaping operation S). This procedure is performed twice, as indicated in Fig.(8). Eventually, $U_{\frac{\pi}{2}}$ is applied a third time. The output is precisely dual rail encoded states of the CNOT operation of the input polariztion states, except for a minus sign (phase shift π) of target output. Thus, correcting for the phase shift and decoding the dual rail states back into polarization states (by the circuit in Fig.(7)) results in the final CNOT output in the polarization states. Formally, the process in the dual rail system is

$$
\begin{array}{ll} CT & \\ (00) & |H\ H\rangle \\ (01) & |H\ V\rangle \\ (10) & |V\ H\rangle \\ (11) & |V\ V\rangle \end{array}
\left.\vphantom{\begin{array}{c}a\\a\\a\\a\end{array}}\right\}
\xrightarrow{(\mathrm{pd})}
\begin{array}{c} a\ \ b\ \ \ \ c \\ \left\{\begin{array}{r} |0\,(0\,1)\,(0\,1)\rangle \\ |0\,(0\,1)\,(1\,0)\rangle \\ |0\,(1\,0)\,(0\,1)\rangle \\ |0\,(1\,0)\,(1\,0)\rangle \end{array}\right\} \end{array}
\xrightarrow{U_{\frac{\pi}{2}}}
\begin{array}{c} a\ \ b\ \ \ \ c \\ \left\{\begin{array}{r} |0\,(0\,1)\,(0\,1)\rangle \\ |0\,(0\,1)\,(1\,0)\rangle \\ |0\,(1\,0)\,(0\,1)\rangle \\ \mathsf{i}\frac{\tilde{\gamma}}{|\tilde{\gamma}|}|1\,(0\,0)\,(0\,0)\rangle \end{array}\right\} \end{array}
\xrightarrow{S}
\begin{array}{c} a\ \ b\ \ \ \ c \\ \left\{\begin{array}{r} |0\,(0\,1)\,(1\,0)\rangle \\ |0\,(0\,1)\,(0\,1)\rangle \\ |0\,(1\,0)\,(1\,0)\rangle \\ \mathsf{i}\frac{\tilde{\gamma}}{|\tilde{\gamma}|}|1\,(0\,0)\,(0\,0)\rangle \end{array}\right\} \end{array} \cdots
$$

$$
\xrightarrow{U_{\frac{\pi}{2}}}
\begin{array}{c} a\ \ b\ \ \ \ c \\ \left\{\begin{array}{r} |0\,(0\,1)\,(1\,0)\rangle \\ |0\,(0\,1)\,(0\,1)\rangle \\ \mathsf{i}\frac{\tilde{\gamma}}{|\tilde{\gamma}|}|1\,(0\,0)\,(0\,0)\rangle \\ -|0\,(1\,0)\,(1\,0)\rangle \end{array}\right\} \end{array}
\xrightarrow{S}
\begin{array}{c} a\ \ b\ \ \ \ c \\ \left\{\begin{array}{r} |0\,(0\,1)\,(0\,1)\rangle \\ |0\,(0\,1)\,(1\,0)\rangle \\ \mathsf{i}\frac{\tilde{\gamma}}{|\tilde{\gamma}|}|1\,(0\,0)\,(0\,0)\rangle \\ -|0\,(1\,0)\,(0\,1)\rangle \end{array}\right\} \end{array}
\xrightarrow{U_{\frac{\pi}{2}}}
\begin{array}{c} a\ \ b\ \ \ \ c \\ \left\{\begin{array}{r} |0\,(0\,1)\,(0\,1)\rangle \\ |0\,(0\,1)\,(1\,0)\rangle \\ -|0\,(1\,0)\,(0\,1)\rangle \\ -|0\,(1\,0)\,(1\,0)\rangle \end{array}\right\} \end{array}
\xrightarrow{(\mathrm{dp})}
\left\{\begin{array}{r} |H\ H\rangle \\ |H\ V\rangle \\ -|V\ V\rangle \\ -|V\ H\rangle \end{array}\right.
\begin{array}{l} CT \\ (00) \\ (01) \\ (11) \\ (10) \end{array}
$$

$$
. \tag{98}
$$

As can be seen from this description, the scalar factor γ in Eq.(98) is automatically compensated. Therefore, fine tuning and normalization of γ are not necessary in this case. The remaining minus signs in the output (with the control qubit equal 1) may be removed by performing a phase shift of π by a single phase gate on mode b either at the input or output point, as indicate in Fig. (8) by the element π.

3.4 Conclusion

We have provided designs of a deterministic quantum CNOT gate which acts as unitary operators on the state spaces of photonic number states as well as polarization states. The deterministic performance is achieved by using the χ^2 nonlinearities which give rise to the Hamiltonian H_1. This Hamiltonian can be realized using the method of coherent photon conversion, as presented in [52]. In particular, the crucial parameters determining the unitary operations can be tuned via the classical electric field used on one mode of CPC system. In the number state design, there are two CPC process and to perform the standard CNOT matrix given in Eq.(33), the phase factors $\frac{\gamma}{|\gamma|}$ and $\frac{\gamma^*}{|\gamma|}$ for the two process have to be made unity. This is tuned by the classical pump field. Alternatively the polarization based CNOT design shows different behaviour scince the aforementioned phase factors disappeared through the process, so the adjustment of the pump field is not required. Since the CNOT gate represents a universal element of logical circuits, their explicit construction can be seen as a proof of concept for the feasibility in principle of general quantum computing devices.

3.5 Future Works

Our future work is to practicaly realize the two CNOT gates in both, polarization and number state bases as well as we hope to serve CPC process in other quatum information processing like, entagled photon generation and in photon detection enhancement.

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الخلاصة

تعتبرالبوابات الكمية من نوع CNOT ,من اللبنات الاساسية للحاسوب الكمي و هي من المصادر الممتازة لربط الوحدات الرقمية الكمية الفوتونية، غير ان النماذج الحالية لتمثيل عمليات CNOT غير كفوءة لتحقيق متطلبات عمليات حسابية فوتونية كمية يمكن مضاعفتها.

ولقد تم هنا ولاول مرة تقديم نموذجين رياضين لمؤثر CNOT وتم التحقق منهما وذلك باختبار دقيق للقواعد المنطقية. كما نقدم في هذا البحث تصميمين لدوائر ضوئية لتمثيل بوابات ضوئية حتمية من نوع CNOT مبنية على الالية الضوئية الكمية لعملية التحويل الضوئي المتشاكه (Coherent photon conversion,CPC) والتي لم تستعمل سابقا الا لبناء بوابات Controlled Z فقط. يتألف التصميم الاول من مركبتين متماثلتين من نوع CPC تعمل بشكل عام بالحالة الكمية المركبة ثلاثية الوحدات الرقمية الكمية موزعة على الوحدة الرقمية الكمية المساعدة ancilla qubit ووحدة الهدف الرقمية الكمية target qubit ووحدة السيطرة الرقمية الكمية control qubit. وهذا المشروع قليل التعقيد من حيث (الدوائر الضوئية) ويمكن بناؤه واقعيا من قواعد الحالات الاساسية العددية الكمية ((Number state basis (|0›,|1› ومن مقطعين متماثلين من نوع CPC مع بعض العناصر البصرية الخطية. والتصميم الثاني هو نموذج حالات المسار المزدوج (dual rail basis)، و هو قليل التعقيد أيضا ومكون من ثلاثة مركبات متماثلة من نوع (CPC) ويمكن بناؤه واقعيا من الحالات الكمية المستقطبة polarization basis (H,V) مع بعض العناصر البصرية الخطية.

.كما ان استخدام الحالة الكمية الضوئية المتشاكهه في الضخ يعتبر عامل قدح سريع للبوابة من شأنه تقليل زمن المعالجة نسبة الى زمن اللاتشاكه.

وزارة التعليم العالي و البحث العلمي

جامعة بغداد

معهد الليزر للدراسات العليا

بوابات CNOT حتمية مقترحة مبنية على التحويل الفوتوني المتشاكه

اطروحة مقدمة

الى معهد الليزر للدراسات العليا/ جامعة بغداد لاستكمال متطلبات نيل درجة الدكتوراة فلسفة في الليزر/ الهندسة الالكترونية والاتصالات

تقدم بها

جواد عبد الكاظم حسن

1433هـ 2012م